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On minimal immersions of R^2 into S^N

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§1. Introduction.

In this paper we treat isometric minimal immersions of the Euclidean 2-plane with a standard flat metric into the unit sphere S^N of the Euclidean space R^{N+1}.

Analytically the problem can be stated as follows. Study a surface Ψ: $R^2 \rightarrow R^{N+1}$ which is given in the form

$$\Psi(x, y)=(\Psi^1(x, y), \cdots, \Psi^{N+1}(x, y))$$

and is defined on the whole plane R^2, and has the following properties: a C^∞-mapping Ψ satisfies on R^2 the equations

$$(\Psi, \Psi)=1, \tag{1.1}$$

$$(\Psi_x, \Psi_x)=(\Psi_y, \Psi_y)=1, \quad (\Psi_x, \Psi_y)=0 \tag{1.2}$$

and

$$\Psi_{xx}+\Psi_{yy}=-2\Psi, \tag{1.3}$$

where (,) denotes the inner product of R^{N+1}. The condition (1.3) under (1.1) tells us that $\Psi(R^2)$ is a minimal surface in S^N.

In the part I of a previous paper [3], the author has proved some formulas for the Laplacians of higher fundamental tensors. In the part II of the above paper, by making use of these results and the complex function theory, we have studied a restricted class of minimal immersions of R^2 into S^N.

In the case of minimal immersions $R^2 \rightarrow S^5$ [4], we have succeeded in a generalization of our previous results.

The purpose of this paper is to give a complete description in the case of any minimal immersions $R^2 \rightarrow S^N$.

If $\Psi(R^2)$ is not contained in a linear subspace of R^{N+1}, N must be an odd integer [2], [3], say $N=2n+1$. Then we shall prove that $\Psi(R^2)$ is an orbit of an abelian Lie subgroup of $SO(2n+2)$ (Theorem 1). From this result and Hsiang's Theorem [1], we know that any isometric minimal immersion of a flat torus into S^{2n+1} must be real algebraic (Theorem 2).