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A ring theoretical proof in a factor category of indecomposable modules

Dedicated to Professor Mutsuo Takahashi on his 60th birthday

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Introduction.

We have widely extended the Krull-Remak-Schmidt-Azumaya's theorem in [4], [5], [6], [7], [8] and [11] and succeeded to prove some of main theorems by virtue of the theory of category ([12], Theorem 20.1 on page 30 and [13], Theorem 2). However, most statements in the above papers are related to modules, but not to categories. Thus, it is natural to expect to be able to prove all results in the frame of ring theory.

Recently, T. Ishii [10] succeeded to prove substantially the implication from i) to ii) in [11] in the frame of ring theory. Hence, the remaining is essentially Theorem 2 in [6].

In this short note, we shall give a ring theoretical proof of the above theorem by making use of an idea given in [10]. First, we shall translate a factor category induced from completely indecomposable modules into a category of semi-simple modules through equivalent functors, which gives a simpler proof of [4], Theorem 7, however it does not work on category of projectives or injectives (see [4] and [6]). Finally, we shall give a ring theoretical proof of [6], Theorem 2 by making great use of results in [10].

§ 1. Factor categories.

Throughout we shall assume that R is a ring with identity and all R-modules are unitary right R-modules. We shall denote the category of all R-modules by $\mathfrak{M}_R$. Let $\{T_\alpha\}_I$ be a set of R-modules. We shall define a full subadditive *category* $\mathfrak{T}$ *induced from* $\{T_\alpha\}_I$ (see [4]). Every objects in $\mathfrak{T}$ consist of all R-modules which are isomorphic to $\sum_K \oplus T_\delta$, where T_δ's are some members in $\{T_\alpha\}_I$ and the set of morphisms coincides with the set of R-homomorphisms. Let $\mathfrak{C}$ be an ideal in $\mathfrak{T}$ (see [2] or [9]). We define the *factor category* $\mathfrak{T}/\mathfrak{C}$ as follows: the objects in $\mathfrak{T}/\mathfrak{C}$ coincide with those in $\mathfrak{T}$ and $[T, T']_{\mathfrak{T}/\mathfrak{C}}=\mathrm{Hom}_R(T, T')/(\mathrm{Hom}_R(T, T')\cap\mathfrak{C})$ for T, T' in $\mathfrak{T}/\mathfrak{C}$.