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# Some results on the fix-points and factorization of entire and meromorphic functions

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## Introduction.

Let $f(z)$ be a complex-valued function of the complex variable $z$. A fix-point $z_0$ of $f$ is a zero of $f(z)-z$. When $f$ is a rational function or an entire function one may define the "iterates" $f_n(z)$ of $f(z)$ by $f_1(z)=f(z)$, $f_n(z)=f_{n-1}(f(z))=f(f_{n-1}(z))$. The existence and distribution of the fix-points play an important role in the theory of iteration of entire functions and the solutions of various functional equations. Regarding the latter subject, we refer the reader to works of Schröeder, Koenigs and others (for a bibliograph cf. [**1**]) which deals with the behavior of the sequence $\{f_n(z)\}$ in the neighborhood of fix-points. As for the former subject, we refer the reader to [**2**] and a book of Kuczma's [**20**]. The theory of iteration of rational functions or entire functions have been extensively studied by Julia [**19**], Rosenbloom [**26**], Fatou [**10**], Myrberg [**21**], Baker [**2**], Cremer [**7, 8**], Töpfer [**27**] and others.

The first interesting result concerning the fix-points and iterates of an entire function was announced by Fatou [**10**] who stated that if any iterate $g_n(z)$ $(n\geqq 2)$ of an entire function $g$ has only a finite number of fix-points, then $g$ is a polynomial. In 1952, Rosenbloom [**25**], based on the technique of Nevanlinna, proved a more general statement that if $f$ and $g$ are entire functions, and if $f$ and $f(g)$ have only a finite number of fix-points, then either $f$ is a polynomial or $g\equiv$constant or $g(z)\equiv z$. In the same paper, the term "prime function" was introduced as an entire function which cannot be represented in the form $f_1(f_2)$, where $f_1$ and $f_2$ are non-linear entire functions. From the definition, we see immediately that every non-linear polynomial of prime degree is prime. However, it is, in general, not easy to tell whether a given entire function is prime or not. For instance, the function $e^z+z$ was stated without proof in [**25**] to be a prime function, and it was stated there that the proof was rather complicated. So far, certain classes of prime functions have been obtained through the investigation of Baker [**4**], Gross [**11, 12, 13, 14, 15**],

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