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On some Galois cohomology groups of a local field and its application to the maximal p-extension

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Introduction.

In this paper we shall prove that some Galois modules of a local field are cohomologically trivial, and as an application of this result we shall give another proof of Šafarevič-Marshall's theorem(1) (see below). This is a generalization of [7], § 8.

Now we formulate our result as follows.

Let k be a complete field of characteristic 0 under a discrete valuation with perfect residue field $\bar{k}$ of characteristic $p\neq 0$ and with absolute ramification order e_k, i.e., $e_k=\mathrm{ord}_k(p)$, where ord_k is the normalized additive valuation of k. Let $\mathcal{F}_k(p)$ be the set of all finite Galois extensions of k of p-power degree contained in the fixed algebraic closure of k and let k_p be the maximal p-extension of k, i.e., the composite field of all fields belonging to $\mathcal{F}_k(p)$. *Fix* a generator σ of the Galois group $G(k_p(\zeta)/k_p)$, where ζ is a primitive p-th root of unity, and let η be the unique element of $\boldsymbol{Z}_p^\times$ such that $\zeta^\sigma=\zeta^\eta$ and $\eta^N=1$, where $N=[k(\zeta):k]$. The group ring $\boldsymbol{Z}_p[G(k_p(\zeta)/k_p)]$ operates on $U_{K(\zeta)}^{(1)}$ for any $K\in\mathcal{F}_k(p)$, hence on $U_{k_p(\zeta)}^{(1)}=\varinjlim U_{K(\zeta)}^{(1)}$ (the inductive limit is taken over all $K\in\mathcal{F}_k(p)$) in the natural way (for the definition of $U_{k(\zeta)}^{(1)}$, see Notations).

DEFINITION. For each $K\in\mathcal{F}_k(p)$, put

$$A(K)=\{x\in U_{K(\zeta)}^{(1)} \mid x^{\sigma-\eta}=1\}$$

and

$$A(k_p)=\{x\in U_{k_p(\zeta)}^{(1)} \mid x^{\sigma-\eta}=1\} .$$

Identifying $G(K/k)$ and $G(K(\zeta)/k(\zeta))$, $A(K)$ becomes $G(K/k)$-module and $A(k_p)$ becomes $G(k_p/k)$-module.

Under the above notations and assumptions we have the following:

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(1) We obtained this independently of Marshall [5]. When I finished to write the manuscript, I knew in Reviews in Number theory Vol. 5 (edited by W. J. Leveque, A. M. S., 1974) that Marshall [5] had already obtained this result, and I rewrited this paper in this form.