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Scattering theory for elliptic systems

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Abstract. We prove existence and completeness of the wave operators and the invariance principle for first order systems even though the perturbation does not have compact support and no unique continuation property is assumed.

§ 1. Introduction.

The systems considered are of the form

$$Hu = E^{-1}\left(\sum_{j=1}^{n} A^j D_j u + Bu\right), \tag{1.1}$$

where u is an m component vector valued function of $x \in E^n$, $A^j(x)$, $E(x)$ and $B(x)$ are $m \times m$ matrix valued measurable functions of x and $D_j = \partial / i\,\partial x_j$. For the unperturbed system we take

$$H_0 u = E_0^{-1} \sum_{j=1}^{n} A_0{}^j D_j u, \tag{1.2}$$

where E_0 and the $A_0{}^j$ are constant matrices. We make the following assumptions:

1. The matrices E_0, $A_0{}^j$, E, A^j are hermitian.
2. H_0 is elliptic and H is uniformly elliptic.
3. E_0 is positive definite and E is uniformly positive definite.
4. The A^j are bounded and uniformly continuous.
5. E is bounded.
6. The distribution derivatives $D_j A^j$ satisfy

$$B - B^* = \sum_{j=1}^{n} D_j A^j, \tag{1.3}$$

where B^* is the hermitian adjoint of B.

7. $B(x)$ is locally square integrable and

$$\sup_x \int_{|x-y|<\delta} |B(y)|^2 |x-y|^{2-n} dy \to 0 \qquad \text{as } \delta \to 0. \tag{1.4}$$