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Tensor products of $C(X)$-spaces and their conjugate spaces

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For any locally compact (Hausdorff) space X, we denote by $C(X)$ and $C_0(X)$ the Banach algebra of all bounded continuous functions on X and the ideal of those $f \in C(X)$ which vanish at infinity, respectively. Thus the conjugate space $C_0(X)'$ of $C_0(X)$ can be identified with the space $M(X)$ of all bounded regular measures on X. Now let $X_1, \cdots, X_N$ be finitely many locally compact spaces, and X the product space thereof. Given a Banach space B, we consider

$$V_0(X) \hat{\otimes} B = C_0(X_1) \hat{\otimes} \cdots \hat{\otimes} C_0(X_N) \hat{\otimes} B ,$$

the (complete) projective tensor product of $C_0(X_1), \cdots, C_0(X_N)$, and B (cf. [**10**]). Notice that the Banach space $V_0(X) \hat{\otimes} B$ can be regarded as a linear subspace of $C(X:B)$, the space of all B-valued bounded continuous functions on X.

The main purpose of this paper is to prove that, under a certain condition on B', the space $(V_0(X) \hat{\otimes} B)'$ has a natural decomposition which is similar to the well-known decomposition $M(X) = M_c(X) + M_d(X)$. As a special case of this result it is shown that $M(X)$ is norm-dense in $V_0(X)'$ if and only if all except at most one X_j are residual (i.e., contain no perfect sets). We also give an application of the latter result to the study of Fourier restriction algebras.

Let $V_0(X) \hat{\otimes} B$ be as above. Then $V_0(X) \hat{\otimes} B$ has a natural Banach $V(X)$-module structure, where $V(X) = C(X_1) \hat{\otimes} \cdots \hat{\otimes} C(X_N) \subset C(X)$:

$$(\phi F)(x) = \phi(x) F(x) \qquad (\phi \in V(X),\ F \in V_0(X) \hat{\otimes} B,\ x \in X) .$$

We define the product $\phi P \in (V_0(X) \hat{\otimes} B)'$ of a $\phi \in V(X)$ and a $P \in (V_0(X) \hat{\otimes} B)'$ by setting

$$\langle F, \phi P \rangle = \langle \phi F, P \rangle \qquad \forall F \in V_0(X) \hat{\otimes} B .$$

Notice that the imbedding $V_0(X) \subset V(X)$ is isometric. We also define the X-support of P, $S_X(P)$, to be the smallest closed subset S of X such that $\langle F, P \rangle = 0$ whenever $F \in V_0(X) \hat{\otimes} B$ and $F = 0$ on some neighborhood of S (cf. [**5**; p. 31]).

DEFINITIONS. Let $P \in (V_0(X) \hat{\otimes} B)'$ be given.

(a) We call P *point-mass-like* if $S_X(P)$ is either a singleton or empty.