J. Math. Soc. Japan
Vol. 27, No. 4, 1975

Reduction theorems for characters of finite groups of Lie type*

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(Received Nov. 27, 1974)

Introduction.

The first main result of this paper is a formula for the values of irreducible complex characters of finite Chevalley groups, of normal or twisted type, on elements whose centralizers are contained in Levi factors of parabolic subgroups. The result is based on the organization, due to Harish-Chandra [**13**] and Springer ([**18**], [**19**]), of the character theory of these groups from the point of view of cusp forms, and can be stated as follows.

THEOREM A. *Let G be a finite group with a split (B, N)-pair of characteristic p, and let (W, R) be the Coxeter system of G. Let ζ be an irreducible complex-valued character of G, such that $(\zeta, \tilde{\varphi}^G)\neq 0$, where φ is an irreducible cuspidal character of some Levi factor L_J of a parabolic subgroup P_J (for $J\subseteqq R$), and $\tilde{\varphi}$ is the extension of φ to P_J with $O_p(P_J)\leqq \ker\tilde{\varphi}$. Let $x\in G$ be an element such that $C_G(x)\leqq L_{J'}$, for some Levi factor $L_{J'}$ of another parabolic subgroup $P_{J'}$. Then $\zeta(x)=0$ unless there exists a subset $J''\subseteqq J'$ such that L_J and $L_{J''}$ are conjugate by an element of the Coxeter group W. If this occurs, then the value $\zeta(x)$ is given by*

$$\zeta(x)=\sum(\zeta, \tilde{\lambda}^G)\lambda(x)\,,$$

where the sum is taken over irreducible characters λ of $L_{J'}$, such that $\lambda\in\tilde{\eta}^{L_J}$, for an irreducible cuspidal character η of $L_{J''}$, with $J''\subseteqq J'$, and $L_{J''}$ conjugate to L_J by an element of W.

A sharper version of Theorem A gives the value of the character $\zeta(x)$ on an element x whose semisimple, or p-regular, part x_s has $L_{J'}$ as its centralizer, in terms of certain decomposition numbers and the values of the characters λ in Theorem A on the unipotent part x_u of x. More precisely,

$$\zeta(x)=\sum\alpha_{\zeta,\lambda}^{x_s}\lambda(x_u)\,,$$

for certain algebraic integers $\alpha_{\zeta,\lambda}^{x_s}$, corresponding to x_s, ζ, and the characters

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* This work was the subject of the author's lecture at the International Symposium in Finite Group Theory, held in Sapporo from Sept. 1-7, 1974 (See [**10**]).

** This research was supported in part by NSF Contract GP 37982 X.