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On embeddings of spaces into ANR and shapes

By Yukihiro KODAMA

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§ 1. Introduction.

Shapes of compact metric spaces were introduced by K. Borsuk [3]. He generalized in [2] and [4] this concept to general metric spaces by defining weak shapes and positions. The notions of shapes or weak shapes of spaces give classifications of spaces coarser than homotopy type and they are determined by circumstance under which the space is embedded into an AR as a closed set. In this paper we shall show that a given metric space X is embedded into an AR with a convenient structure for investigating shape theoretical properties of X. By making use of this embedding, for a locally compact metric space X, it is shown that there is a locally compact Δ-space whose weak shape is equal to X's. In case X is compact this fact has been proved in [12] by Mardešić-Segal approach to shape [13]. However the compactness of a space is essential in Mardešić-Segal approach and we can not use it for our case. The concept of fundamental skeletons of a space is introduced. Every Δ-space has fundamental skeletons, but it is known that there is an AR which does not have fundamental skeletons. Finally a partial answer to a problem concerning position raised by Borsuk [4] is given.

Throughout this paper all of spaces are metric and maps are continuous. By an AR and an ANR we mean always an AR for metric spaces and an ANR for metric spaces, and by dimension we imply the covering dimension.

§ 2. Embedding of spaces into ANR.

Let X and Y be metric spaces and let $f: X \to Y$ be a continuous map. We define a *metrizable mapping cylinder* $M(X, Y, f)$ as follows. It is obtained by identifying points $(x, 1) \in X \times \{1\} \subset X \times I$ and $f(x) \in Y$ for $x \in X$ in a topological sum $X \times I \cup Y$, where $I = [0, 1]$. Let $p: X \times I \cup Y \to M(X, Y, f)$ be a quotient map. We denote $p(x, t)$ for $(x, t) \in X \times I$ by (x, t) and $p(y)$ for $y \in Y$ by y simply. We consider X and Y as subsets of $M(X, Y, f)$ (X is identified with the set $\{(x, 0): x \in X\}$). We give $M(X, Y, f)$ the following topology. A point (x, t), $x \in X$ and $0 \leqq t < 1$, has a neighborhood system consisting of all sets of the form $U \times V$, where U and V range over neighborhoods of x and t