J. Math. Soc. Japan
Vol. 27, No. 4, 1975

Real hypersurfaces in a complex projective space with constant principal curvatures II

By Ryoichi TAKAGI

(Received July 4, 1974)

Introduction.

Let $P_n(\boldsymbol{C})$ be a complex projective space of complex dimension n ($\geqq 2$) with the metric of constant holomorphic sectional curvature. We proved in [3] that if M is a connected complete real hypersurface in $P_n(\boldsymbol{C})$ with two constant principal curvatures then M is a geodesic hypersphere. The purpose of this paper is to determine all real hypersurfaces in $P_n(\boldsymbol{C})$ ($n \geqq 3$) with three constant principal curvatures.

To state our result we begin with examples of real hypersurfaces in $P_n(\boldsymbol{C})$ with three constant principal curvatures. Let $\boldsymbol{C}^{n+1}$ be the space of $(n+1)$-tuples of complex numbers $(z_1, \cdots, z_{n+1})$, and π be the canonical projection of $\boldsymbol{C}^{n+1}-\{0\}$ onto $P_n(\boldsymbol{C})$. For an integer m ($2 \leqq m \leqq n-1$) and a positive number s we denote by $M'(2n, m, s)$ a real hypersurface in $\boldsymbol{C}^{n+1}$ defined by

$$\sum_{j=1}^{m} |z_j|^2 = s \sum_{j=m+1}^{n+1} |z_j|^2, \qquad (z_1, \cdots, z_{n+1}) \neq 0 .$$

For a number t ($0<t<1$) we denote by $M'(2n, t)$ a real hypersurface in $\boldsymbol{C}^{n+1}$ defined by

$$|\sum_{j=1}^{n+1} z_j^2|^2 = t(\sum_{j=1}^{n+1} |z_j|^2)^2, \qquad (z_1, \cdots, z_{n+1}) \neq 0 .$$

It will be shown that $M(2n-1, m, s)=\pi(M'(2n, m, s))$ ($n \geqq 3$) and $M(2n-1, t) = \pi(M'(2n, t))$ ($n \geqq 2$) are connected compact real hypersurfaces in $P_n(\boldsymbol{C})$ with three constant principal curvatures.

MAIN THEOREM. *If M is a connected complete real hypersurface in $P_n(\boldsymbol{C})$ ($n \geqq 3$) with three constant principal curvatures, then M is congruent to some $M(2n-1, m, s)$ or to some $M(2n-1, t)$, i.e., there exists an isometry g of $P_n(\boldsymbol{C})$ such that $g(M)=M(2n-1, m, s)$ or $g(M)=M(2n-1, t)$.*

In §1 we shall study general properties of a real hypersurface M in $P_n(\boldsymbol{C})$ with constant principal curvatures. In §3, on the assumption that M has three constant principal curvatures, we shall give equations which the almost contact structure of M must satisfy, which are summed up as Lemma 3.4.