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On Veronese manifolds

By Takehiro ITOH(*)

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From differential geometric point of view, a Veronese surface may be considered as a minimal immersion of a 2-dimensional sphere of curvature 1/3 into a 4-dimensional unit sphere. S. S. Chern, M. doCarmo and S. Kobayashi [**1**] gave a local characterization of a Veronese surface. The author [**4**] also characterized a Veronese surface by non-zero constant normal curvature.

We call an isometric immersion (a submanifold) an *isotropic immersion* (*isotropic*) if all its normal curvature vectors have the same length at each point. Let $P^n(c)$ (resp. $P_n(c)$) be an n-dimensional real (resp. complex) projective space of curvature c and $S^m(c)$ be an m-dimensional sphere of curvature c. B. O'Neill [**10**] proved the following results:

(A) *There exists a non-umbilic isotropic minimal imbedding* $\psi: P^n(c) \to S^{n+p}(\tilde{c})$ *where* $c=\dfrac{n\tilde{c}}{2(n+1)}$ *and* $p=\dfrac{1}{2}n(n+1)-1$.

(B) *There exists a Kaehler imbedding* $\psi: P_n(c) \to P_{n+p}(\tilde{c})$, *where* $2c=\tilde{c}$ *and* $p=\dfrac{1}{2}n(n+1)$.

Taking account of [8], we may call these submanifolds *the Veronese submanifolds*, in particular, we may call the former *the real Veronese submanifold* and the latter *the complex Veronese submanifold*. M. doCarmo and N. Wallach [**2**] characterized a real Veronese submanifold. The author and K. Ogiue ([**6**], [**7**]) also gave some characterizations of a real Veronese submanifold in terms of isotropic immersion. K. Ogiue [**9**] gave characterizations of a complex Veronese submanifold.

The purpose of the present paper is to characterize Veronese manifolds by means of geometric invariant functions on submanifolds.

§ 1. Real submanifolds in real space forms.

Let M^n be an n-dimensional submanifold immersed in an $(n+p)$-dimensional Riemannian manifold $\tilde{M}^{n+p}$ of constant curvature $\tilde{c}$, (i. e., Riemannian submani-

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