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Homeomorphism between the open unit disk and a Gleason part

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§1. Introduction.

Let $P(m)$ be the non-trivial Gleason part which contains a complex homomorphism m of a uniform algebra A on a compact space X, and suppose that m has a unique positive representing measure on X (for the definitions see §2). Then, it is known that there is a one-one continuous map τ of the open unit disk D in the complex plane onto $P(m)$ (in the Gelfand topology) such that for every $f\in A$, $\tau(t)(f)$ is analytic in D (Wermer's embedding theorem). But τ is not necessarily a homeomorphism. Such examples are found in Wermer [**10**], p. 443, Hoffman [**6**], p. 109 and others. The purpose of this paper is to establish some conditions for τ to be a homeomorphism. In §2 some preliminaries are given. In §3 we state and prove our results, and an example is studied in relation to our main Theorem 3.2.

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§2. Preliminaries.

For a commutative Banach algebra B over the complex numbers, let $\mathcal{M}(B)$ be the maximal ideal space (or the space of complex homomorphisms) of B which has the Gelfand topology, and let $\hat{f}$ be the Gelfand transform of $f\in B$.

Let $C(X)$ be the algebra of all complex-valued continuous functions on a compact Hausdorff space X and let A be a *uniform algebra* on X, that is, a closed (by supremum norm $\|\ \|$) subalgebra in $C(X)$ containing constants and separating points of X. For φ in $\mathcal{M}(A)$, $M_\varphi=M_\varphi(A)$ denotes the set of representing measures on X for φ, i.e., the set of all probability measures μ on X such that $\varphi(f)=\int fd\mu$ for all $f\in A$.

Given φ and θ in $\mathcal{M}(A)$, we set

$$\sigma(\varphi,\theta)=\sup\{|\varphi(f)|: f\in A,\ \|f\|\leqq 1,\ \theta(f)=0\}, \tag{2.1}$$

and write $\varphi\sim\theta$ if and only if $\sigma(\varphi,\theta)<1$. Then $\sim$ is an equivalence relation