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On the prolongation of local holomorphic solutions of nonlinear partial differential equations

By Yoshimichi TSUNO*)

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§ 1. Introduction.

One of important problems in the theory of partial differential equations in the complex domain is the following: When can the holomorphic solutions of a partial differential equation defined in some domain be continued to a larger domain? For linear partial differential equations this question has been answered by several authors. (See e.g. [3] and the references quoted therein.) One of the main results regarding this question is a theorem of M. Zerner [5] which states that the solutions of a linear partial differential equation can be continued holomorphically over any non-characteristic hypersurfaces.

In this paper we are concerned with holomorphic continuation of solutions of general nonlinear partial differential equations in the complex domain. Our purpose here is to present a continuation theorem which corresponds to that of Zerner [5] for linear equations. It can be stated as follows: If the tangent plane at a boundary point of the domain in which the solutions are defined is non-characteristic *for any Cauchy data*, then every *bounded* solution can be holomorphic near that point. Our argument depends on the quantitative property of the domain in which the solutions of the Cauchy problem become holomorphic. In § 2, following carefully the well-known proof ([1], [2]) of the Cauchy-Kowalewsky theorem, we obtain the desired result. Then in § 3 we find the continuation theorem. In the last section, § 4, we study the single equation of the first order with two independent variables for which the assumptions of the results in section 3 are not satisfied. In this case we can construct a solution which cannot be prolonged under some conditions on the boundary of the domain and the characteristic curve.

The results of this paper was already announced in [4] without proofs.

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