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On fixed point free $SO(3)$-actions on homotopy 7-spheres

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§ 0. Introduction.

Let $SO(3)$ be the rotation group (see § 1). In this paper, we shall study smooth $SO(3)$-actions on homotopy 7-spheres without fixed points. Our category is the smooth category. In [5], we have studied some $SO(3)$-actions on homotopy 7-spheres, mainly in the case with two or three orbit types. In that case, the actions have fixed points (see [5]). Our present paper is concerned with the case without fixed points.

Let α and β be the real irreducible representations of $SO(3)$ of dimension 3 and 5 respectively (see § 1). Then $\alpha\oplus\beta$ induces a linear action of $SO(3)$ on the 7-sphere S^7. A simple observation shows that this is the only linear action on S^7 which has no fixed points. Let (Σ^7, φ) be a smooth $SO(3)$-action on a homotopy 7-sphere Σ^7 (here φ; $SO(3)\times\Sigma^7\to\Sigma^7$ is a smooth map defining the action). For $g\in SO(3)$ and $x\in\Sigma^7$, gx denotes $\varphi(g, x)$. The isotropy subgroup of x, G_x, is defined by $G_x=\{g\in SO(3)\,|\,gx=x\}$. Then the set of the conjugacy classes $\{(G_x)\,|\,x\in\Sigma^7\}$ is called as the isotropy subgroup type of (Σ^7, φ). Now we assume that (Σ^7, φ) is fixed point free, that is, for each $x\in\Sigma^7$, G_x is a proper subgroup of $SO(3)$. Then we ask if the isotropy subgroup type of (Σ^7, φ) coincides with that of the linear action $\alpha\oplus\beta$. The answer is given by the following two theorems.

THEOREM I. *Let (Σ^7, φ) be a smooth $SO(3)$-action on a homotopy 7-sphere Σ^7 without fixed points. Then the isotropy subgroup type of (Σ^7, φ) is one of the following two types,*

(a) $\{(e), (Z_2), (D_2), (SO(2)), (N)\}$ *and*

(b) $\{(e), (Z_2), (D_2), (SO(2)), (N), (Z_{2k+1}), (D_{2k+1})\}$ (*k is a positive integer*),

(*For the notations see* § 1).

The type (a) in the above theorem is that of the linear action $\alpha\oplus\beta$ (§ 2). There is no linear action having (b) as its isotropy subgroup type.

THEOREM II. *For each positive integer k, there is a smooth $SO(3)$-action on the standard 7-sphere S^7 with isotropy subgroup type* (b) *of Theorem* I.

Theorem I will be proved in § 3 and Theorem II in § 4.