J. Math. Soc. Japan
Vol. 27, No. 3, 1975

A test of Picard principle for rotation free densities

By Mitsuru NAKAI

(Received June 4, 1974)

Consider a nonnegative locally Hölder continuous function $P(z)$ on the punctured unit disk $0<|z|\leqq 1$. Such a function will be referred to as a *density*. To describe the potential theoretic singular behavior of a density $P(z)$ at $z=0$ we consider the *elliptic dimension*, $\dim P$ in notation, which is defined to be the dimension of the half module of nonnegative solutions of the equation $\Delta u=Pu$ on $0<|z|<1$ having the continuous boundary values zero on $|z|=1$. After Bouligand we say that the *Picard principle* is valid for a density P at $z=0$ if $\dim P=1$. One of the central theme of the study of elliptic dimensions is to determine the range of the mapping $P\to\dim P$ and in particular to determine $\{P;\dim P=1\}$, the family of densities for which the Picard principle is valid. Although we have various results on this subject obtained by many authors listed in the references at the end of this paper, the study is quite far from being complete. As an experimental study we considered in our former paper [**13**] *rotation free* densities $P(z)$ in the sense that $P(z)=P(|z|)$ for every z in $0<|z|\leqq 1$. We showed that the Martin compactification Ω_P^* of the punctured open unit disk $\Omega: 0<|z|<1$ with respect to a rotation free density P on $0<|z|\leqq 1$ is homeomorphic to a closed annulus, i.e.

$$\Omega_P^*\approx(\alpha(P)\leqq|z|\leqq 1) \tag{1}$$

where $\alpha(P)$, referred to as the *singularity index* of P, is the proper quantity in $[0, 1)$ associated with P determined as follows: The ordinary differential equation

$$(ru'(r))'=r(P(r)+j^2/r^2)u(r)$$

has a unique bounded solution $e_j(r)$ on $(0, 1]$ with the initial condition $e_j(1)=1$ $(j=0, 1)$ and the function $e_1(r)/e_0(r)$ has a limit as $r\to 0$ which is defined to be $\alpha(P)$:

$$\alpha(P)=\lim_{r\to 0}e_1(r)/e_0(r)\in[0, 1)\,.$$

Therefore in particular we have

$$\dim P=1+\alpha(P)\cdot\mathfrak{c} \tag{2}$$

where $\mathfrak{c}$ is the cardinal number of continuum, and there really can occur both