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Micro-hyperbolic pseudo-differential operators I

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§ 0. Introduction.

Bony and Schapira [1] has proved that the Cauchy problem is well-posed for hyperbolic operators with variable coefficients in the framework of hyperfunctions. In their paper they took up the defining functions of hyperfunctions, and by applying their refined version of the Cauchy-Kovalevsky theorem they proved that the solutions with the initial data which are the defining functions of hyperfunction data become also the defining functions of sought for hyperfunction solutions. In this paper, their results are extended to the case of micro-hyperbolic pseudo-differential operators; namely we will prove that in the framework of microfunctions the Cauchy problem is well-posed for the pseudo-differential operators of that type. This result implies the result of Bony and Schapira about the hyperbolic differential operators.

The essential step in our argument is in the construction of the elementary solution for the Cauchy problem. As a by-product of this method, we obtain a rather wide class of solvable pseudo-differential operators, whose null solutions propagate in one-sided direction along the "bicharacteristics" (if it exists).

Generalization of the results in this paper to the system of pseudo-differential equations will be dealt with in the subsequent paper.

A pseudo-differential operator $P(x, D_x)$ is said to be partially micro-hyperbolic at $(x_0, \sqrt{-1}\xi_0)$ with respect to the direction $\langle\vartheta, dx\rangle+\langle\rho, d\xi\rangle$ (see § 1 for the precise definition) if $P_m(x+\sqrt{-1}\varepsilon\rho, \sqrt{-1}\xi+\varepsilon\vartheta)$ never vanishes for every (x, ξ) near (x_0, ξ_0) and $0<\varepsilon\ll 1$. We reduce it, by means of a quantized contact transformation, to an operator of the form $P=D_1-A(x, D')$ where A is a matrix of pseudo-differential operators of order $\leqq 1$ commuting with x_1 and such that all the eigenvalues of the principal symbol $A_1(x, \sqrt{-1}\xi')$ have non-negative real part. Then we construct a formal solution $G(x, D')=\sum_\alpha a_\alpha(x)D'^\alpha$ such that $PG=0$ and $G|_{x_1=0}=1$ in § 2. In § 3 and § 4, we will show that GD_1^{-1} can be realized as a microfunction, which becomes an elementary solution of P. In this way, we obtain the following theorems (Theorem 5.2 and Theorem 5.5).

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