J. Math. Soc. Japan
Vol. 27, No. 2, 1975

On cohomology mod 2 of the classifying spaces of non-simply connected classical Lie groups

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(Received May 31, 1974)

§1. Introduction.

Let $\boldsymbol{G}$ be a compact connected Lie group. Consider the following two statements:

(1.1) $H^*(\boldsymbol{G}; \boldsymbol{Z}_2)$ has a simple system of primitive generators.

(1.2) $H^*(B\boldsymbol{G}; \boldsymbol{Z}_2)$ is a polynomial algebra.

As is well known, (1.2) implies (1.1). On the other hand for any simple $\boldsymbol{G}$ except $\boldsymbol{Spin}(2^k+1)$, $k \geqq 4$, and $\boldsymbol{PSp}(2n+1)$, (1.1) implies (1.2).

According to Borel [4] $\boldsymbol{Spin}(2^k+1)$, $k \geqq 4$, does not satisfy (1.2).

In this paper, we show (1.2) for $\boldsymbol{PSp}(2n+1)$ namely $H^*(B\boldsymbol{PSp}(2n+1); \boldsymbol{Z}_2)$ is a polynomial algebra (see Theorem 4.4).

In the section 2 we consider some 2-groups in $\boldsymbol{Sp}(n)$ and $\boldsymbol{PSp}(n)$. In the section 3 we consider a kind of stability in the cohomology $H^*(B\boldsymbol{PSp}(n); \boldsymbol{Z}_2)$ and $H^*(\boldsymbol{PSp}(n); \boldsymbol{Z}_2)$. The section 4 is devoted to the determination of $H^*(B\boldsymbol{PSp}(2n+1); \boldsymbol{Z}_2)$. In the section 5 we can classify the compact connected simple Lie groups whose mod 2 cohomology rings have simple systems of universally transgressive generators (Theorem 5.2).

Throughout this paper the map $B\boldsymbol{G} \to B\boldsymbol{H}$ induced by a homomorphism $f: \boldsymbol{G} \to \boldsymbol{H}$ of Lie groups is also denoted by the symbol f (Milnor [8]).

The author would like to thank Prof. H. Toda for his comments during the preparation of the manuscript.

§2. Some 2-groups in $\boldsymbol{Sp}(n)$ and $\boldsymbol{PSp}(n)$.

In this section various finite subgroups of $\boldsymbol{Sp}(n)$ and $\boldsymbol{PSp}(n)$ are considered. We use the following notations

$$\begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix} = (a_1, \cdots, a_n) \in \boldsymbol{Sp}(n) \qquad \text{for} \quad a_i \in \boldsymbol{Sp}(1).$$