J. Math. Soc. Japan
Vol. 27, No. 2, 1975

On flat over-rings of a Krull domain

By Ken-ichi YOSHIDA

(Received Jan. 23, 1974)

Introduction.

Let A be an integral domain and let K be the quotient field of A. In this paper we are mainly concerned with a subring B of K containing A. For the sake of simplicity we shall call such an intermediate ring an over ring of A hereafter. The purpose of this paper is to study the relationship between an over ring B and subsets $F_A(B)$ and $F_A^*(B)$ of Spec A defined by

$$F_A(B)=\{\mathfrak{p}\in \mathrm{Spec}\,A\,;\; A_{\mathfrak{p}}\subsetneqq B\otimes_A A_{\mathfrak{p}}=B_{\mathfrak{p}}\}$$

and

$$F_A^*(B)=\{\mathfrak{p}\in F_A(B)\,;\ \mathrm{height}\,\mathfrak{p}=1\}$$

respectively. Among others it will be shown that if A is a Krull domain and B is a flat over-domain of A, then B is determined uniquely by $F_A^*(B)$. Moreover if B is a flat over-domain of A, B is finitely generated over A if and only if $F_A^*(B)$ is a finite set.

Following the usual terminology, rings are always understood to be commutative and to have the identity elements. For a ring A, Spec A stands for the set of all prime ideals of A and $\mathrm{Ht}_1(A)$ is the set of all prime ideals of A with height 1.

§ 1. On $F_A(B)$.

The following well-known fact will be used frequently in this paper, so we write down it as a lemma without proof (cf. [3]).

(1.1) LEMMA. *Let A be a ring and B an A-algebra contained in the total quotient ring of A. Then the following four conditions are equivalent to each other:*

(1) *B is flat over A.*

(2) *$B_{\mathfrak{p}}=B\otimes_A A_{\mathfrak{p}}$ is flat over $A_{\mathfrak{p}}$ for any $\mathfrak{p}\in \mathrm{Spec}\,A$.*

(3) *$A_{A\cap\mathfrak{P}}=B_{\mathfrak{P}}$ for any $\mathfrak{P}\in \mathrm{Spec}\,B$.*

(4) *For every $\mathfrak{p}\in \mathrm{Spec}\,A$, either $\mathfrak{p}B=B$ or $A_{\mathfrak{p}}=B_{\mathfrak{p}}$.*

Let A be an integral domain and let B be an over-ring of A. We shall introduce the sets: