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On vanishing of cohomology attached to certain many valued meromorphic functions

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1. Let V be a compact Kähler manifold of the complex dimension n. Let $D=\sum\lambda_k\Gamma_k$ ($\lambda_k\in\boldsymbol{C}$) be a bounding $(2n-2)$-cycle on V consisting of a finite number of irreducible closed analytic divisors Γ_k. Let $F_D(z)=\exp\Phi_D(z)$ be a multiplicative meromorphic function having D as its divisor. It is known that this function is determined uniquely up to a multiplicative constant (see [**7**]). Let ρ_λ be the scalar representation of the fundamental group G of $V-|D|$ into $\boldsymbol{C}^*$ canonically induced by the function $F_D(z)$, where $|D|$ denotes the polyhedron representing D. We denote by $\mathcal{S}_\lambda$ the local system over $\boldsymbol{C}$ defined by ρ_λ. Let $T(|D|)$ be a small tubular neighbourhood of $|D|$ in V associated with Whitney stratification of $|D|$ constructed by R. Thom (see [**13**] Théorème 1.D.1, page 248). We make the three following assumptions:

I. *The cohomology with local coefficients* $\mathcal{S}_\lambda$, $H^p(T(|D|)-|D|, \mathcal{S}_\lambda)$ *on* $T(|D|)-|D|$ *vanish for all* $p\geqq 0$.

II. *The critical points of* $\mathrm{Re}\,\Phi_D(z)$ *are all isolated and non-degenerate on* $V-|D|$.

III. *There exists a complete Kähler metric* $(ds)^2$ *on* $V-|D|$ *so that* $V-|D|$ *becomes a symplectic manifold.*

Then we have the following theorem:

THEOREM 1. $H^p(V-|D|, \mathcal{S}_\lambda)=(0)$ *for* $p\neq n$.

This theorem was stated in [**1**] for special cases. See also [**6**].

2. For the proof of the preceding theorem we make use of Morse theory. Let $\sum g_{i\bar{j}}dz^i\cdot d\bar{z}^j$ be a local expression of the Kähler metric $(ds)^2$ of $V-|D|$ with respect to local coordinates $(z^1, z^2, \cdots, z^n)$ of V. We consider a vector field on $V-|D|$ of the following form:

$$dz^i/dt=\sum_j(\mathrm{Re}\,\Phi_D/\partial\bar{z}^j)\cdot g^{i\bar{j}}. \tag{1}$$

It can be verified that this expression is independent of the choice of local coordinates and so defines a well defined vector field X on $V-|D|$. This vector field is closely related to the symplectic structure on $V-|D|$. Consider the following 2-form ω on $(V-|D|)\times\boldsymbol{R}$: