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Ergodicity and capacity of information channels with noise sources[1)]

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§1. Introduction.

The information channel is defined as a sort of conditional distributions on a direct product space of an input alphabet space and an output alphabet space, which are direct product spaces of countable copies of finite sets, conditioned by a Borel field of an input alphabet space (cf. Feinstein [3] and Hinchin [5]). The channel defined in this way is the most abstract and general one. However, many actual communication channels are imagined to have noise sources. The channel of additive noise is a typical one of such cases.

In this paper, we shall clarify a relation between ergodicity of such a channel and that of its noise source, and study about the channel capacity for these channels.

§2. Preliminary.

Let $(X, \mathcal{X})$ and $(Y, \mathcal{Y})$ be measurable spaces with measurable transformations S, T on X and Y respectively, and Π be a set of S-invariant probability measures on X. Assume Π to be non-empty. An element p in Π is called *the input source. The channel* (from X to Y) is a numerical function ν on $X\times\mathcal{Y}$ which satisfies the followings:

(i) for any $x\in X$, $\nu_x(\cdot)$ is a probability measure on $\mathcal{Y}$,

(ii) for any $F\in\mathcal{Y}$, $\nu_\cdot(F)$ is a measurable function on X,

and

(iii) $\nu_{Sx}(F)=\nu_x(T^{-1}F)$ for any $x\in X$ and $F\in\mathcal{Y}$.

An output source q derived from an input source p and a channel ν is defined by

$$q(F)=\int_X \nu_x(F)p(dx) \qquad (F\in\mathcal{Y})$$

and denoted by $q(\cdot)=q(\cdot\,;p,\nu)$, which is a T-invariant probability measure on Y. *A compound source* r derived from an input source p and a channel ν is defined by

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