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Ergodic theorems for contraction semi-groups

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§ 1. Introduction.

The main purpose of this note is to introduce the absolute value of a strongly continuous one-parameter semi-group of contractions on $L_1(X)$, which is again a semi-group and to prove the local ergodic theorem and the ratio ergodic theorem by making use of the introduced semi-group. The absolute value of a bounded linear operator on $L_1(X)$ which is bounded also on $L_\infty(X)$ was introduced by N. Dunford and J. Schwartz [7]. The result was generalized by R. Chacon and U. Krengel [6] as described in Lemma 1 of the present note. But, as Krengel [10] remarked, an essentially same result was obtained much earlier by Kantrovič [8]. We shall introduce the absolute value of a contraction semi-group (Theorem 1). The local ergodic theorem for positive contraction semi-groups on $L_1(X)$ was conjectured by U. Krengel and proved by U. Krengel [9] and D. Ornstein [13] independently. M. Akcoglu and R. Chacon [2] and T. Terrell [14, 15] gave different treatments of the theorem. D. Ornstein [13] gave a proof of the theorem for a contraction semi-group on $L_1(X)$ which is a contraction semi-group also on $L_\infty(X)$. T. Terrell [14] independently proved the theorem for an n-parameter contraction semi-group on $L_1(X)$ which is a contraction semi-group also on $L_\infty(X)$. We shall generalize Ornstein's theorem and prove the local ergodic theorem for a contraction semi-group (T_t) (Theorem 2) by making use of the absolute value of the semi-group (T_t). Further we shall prove a ratio ergodic theorem for a contraction semi-group (Theorem 3). This is a continuous version of Chacon's ratio ergodic theorem for a contraction T and a T-admissible sequence [5].

§ 2. Definitions and theorems.

Let $(X, \mathfrak{B}, m)$ be a σ-finite measure space and $L_1(X)=L_1(X, \mathfrak{B}, m)$ the Banach space of complex-valued integrable functions on X. Let (T_t) $(t \geqq 0)$ be a strongly continuous one-parameter semi-group of linear contractions on $L_1(X)$. In the sequel we call such a semi-group a contraction semi-group. This means that

(A) T_t is a linear operator on $L_1(X)$ such that $\|T_t\| \leqq 1$ for any $t \geqq 0$