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Manifolds with vanishing Weyl or Bochner curvature tensor

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§ 1. Introduction.

Let M be a Riemannian manifold of dimension $n>3$ and denote by g_{ji}, $K_{kji}{}^{h}$, K_{ji} and K the metric tensor, the curvature tensor, the Ricci tensor and the scalar curvature of M respectively.

If M is locally conformal to a Euclidean space then M is said to be conformally flat. For a conformally flat M, the Weyl conformal curvature tensor given by

$$C_{kji}{}^{h}=K_{kji}{}^{h}+\delta_{k}^{h}C_{ji}-\delta_{j}^{h}C_{ki}+C_{k}{}^{h}g_{ji}-C_{j}{}^{h}g_{ki} \tag{1.1}$$

vanishes identically, where

$$C_{ji}=-\frac{1}{n-2}K_{ji}+\frac{1}{2(n-1)(n-2)}Kg_{ji}, \qquad C_{k}{}^{h}=C_{kt}g^{th}, \tag{1.2}$$

g^{th} being contravariant components of the metric tensor. Conversely if $C_{kji}{}^{h}$ vanishes identically, then M is conformally flat [3], [6].

One of the purposes of the present paper is to prove the following:

THEOREM 1. *In order that a Riemannian manifold of dimension $n>3$ is conformally flat, it is necessary and sufficient that there exists a (unique) quadratic form Q on the manifold such that the sectional curvature $K(\sigma)$ with respect to a section σ is the trace of the restriction of Q to σ, i.e. $K(\sigma)=\operatorname{trace} Q/\sigma$, the metric being also restricted to σ.*

Let M be an n-dimensional Kaehlerian manifold and denote by g_{ji}, $F_{i}{}^{h}$, $K_{kji}{}^{h}$, K_{ji} and K the metric tensor, the complex structure tensor, the curvature tensor, the Ricci tensor and the scalar curvature of M respectively. Bochner [1] (see also [4], [9]) introduced a curvature tensor given by

$$\begin{aligned} B_{kji}{}^{h}=&K_{kji}{}^{h}+\delta_{k}^{h}L_{ji}-\delta_{j}^{h}L_{ki}+L_{k}{}^{h}g_{ji}-L_{j}{}^{h}g_{ki}\\ &+F_{k}{}^{h}M_{ji}-F_{j}{}^{h}M_{ki}+M_{k}{}^{h}F_{ji}-M_{j}{}^{h}F_{ki}-2(M_{kj}F_{i}{}^{h}+F_{kj}M_{i}{}^{h}), \end{aligned} \tag{1.3}$$

where

$$L_{ji}=-\frac{1}{n+4}K_{ji}+\frac{1}{2(n+2)(n+4)}Kg_{ji}, \qquad L_{k}{}^{h}=L_{kt}g^{th},$$

$$M_{ji}=-L_{jt}F_{i}{}^{t}, \qquad M_{k}{}^{h}=M_{kt}g^{th}$$