J. Math. Soc. Japan
Vol. 27, No. 1, 1975

Point derivations on commutative Banach algebras and estimates of the $A(X)$-metric norm

By Mikihiro HAYASHI

(Received May 28, 1973)

§ 1. Introduction.

Let A be a commutative Banach algebra with unit 1. We denote the set of all homomorphisms of A onto $\boldsymbol{C}$ by $\mathcal{M}(A)$, which is called the maximal ideal space of A. For $\phi\in\mathcal{M}(A)$, a point derivation on A at ϕ is an (algebraic) linear functional D on A with the property that $D(fg)=\phi(f)D(g)+\phi(g)D(f)$ for all $f, g\in A$. In this paper we consider the point derivations which are defined as follows. Let $\hat{f}(\phi)=\phi(f)$ be the Gelfand transform and let $\{\phi_r, t_r\}$ be a pair of nets in $\mathcal{M}(A)\times\boldsymbol{C}\backslash\{0\}$ with the following properties:

(1.1) ϕ_r converges to ϕ in $\mathcal{M}(A)$ with the weak*-topology,

(1.2) t_r converges to 0 in $\boldsymbol{C}$,

(1.3) $\dfrac{\hat{f}(\phi_r)-\hat{f}(\phi)}{t_r}$ converges for any $f\in A$.

Then the limit $D(f)=\lim_r \dfrac{\hat{f}(\phi_r)-\hat{f}(\phi)}{t_r}$ defines a point derivation at ϕ.

In section 2 considering this kind of point derivation we shall give an another proof of Browder's theorem; there exists a nonzero point derivation at ϕ if ϕ is not isolated in $\mathcal{M}(A)$ with the metric topology. Also we shall prove that there exists a nonzero continuous point derivation at ϕ if ϕ is not isolated in $\mathcal{M}(A)$ with the metric topology and the norm $\|\psi-\phi\|$ of the metric topology is equivalent to a semi-metric $|\psi(w_1)-\phi(w_1)|+\cdots+|\psi(w_n)-\phi(w_n)|$ of the weak* topology in some metric neighborhood of ϕ in $\mathcal{M}(A)$, where $w_1, \cdots, w_n\in A$.

In the remaining sections we shall consider the function algebra $A(X)$ on a compact plane set X. In this case we obtain more exact results. As is well known, these results are translated for the case $R(X)$ and the proofs for the case $R(X)$ are performed similarly. We state here the corresponding results for $R(X)$. Let $R_0(X)$ be the set of all rational functions with poles off X. $R(X)$ is the uniform closure of $R_0(X)$ on X. The maximal ideal space of $R(X)$ is identified with X. It is known that each of the following condi-