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General convergence theorems for the numerical function and the nonstationary stochastic processes

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§ 1. Introduction.

We shall discuss the validity of the limit relation

$$\lim_{n\to\infty} n\int_{-\infty}^{\infty} f(x+u)K(nu)du = f(x)\int_{-\infty}^{\infty} K(u)du .$$

A theorem concerning this relation was given in S. Bochner [1] and is well known. It was generalized by S. Bochner and S. Izumi [2], S. Izumi [3]. The corresponding theorem for the stochastic process was obtained by T. Kawata [4], [5]. In the present paper we shall deal with the generalizations of these theorems.

§ 2. N-functions.

Known definitions and results which we are going to use in this paper are given. An N-function $M(u)$ admits the representation

$$M(u) = \int_0^{|u|} p(t)dt ,$$

where the function $p(t)$ is right-continuous for $t \geqq 0$, positive for $t > 0$, non-decreasing, and satisfies the conditions

$$p(0) = 0 , \qquad p(\infty) = \lim_{t\to\infty} p(t) = \infty .$$

Let $q(s) = \sup_{p(t)\leqq s} t$, $(s \geqq 0)$. Then

$$M(u) = \int_0^{|u|} p(t)dt , \qquad N(v) = \int_0^{|v|} q(s)ds$$

are called mutually complementary N-functions. Let $M(u)$ be an N-function. We shall denote by $L_M(G)$, where G denotes a bounded (unbounded) set in a finite-dimensional Euclidean space, the class of real-valued functions $u(x)$, defined on G, for which