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On deformations of holomorphic maps II

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This paper is the second part of a study of deformations of holomorphic maps. In the first part [4], which will be referred to as Part I, we have proved two fundamental theorems on deformations of non-degenerate holomorphic maps. In § 4, we shall generalize these two theorems to the case in which the holomorphic maps in consideration are not necessarily non-degenerate.

In §§ 5, 6, we shall study deformations of holomorphic maps in the sense iii) in the introduction of Part I. Namely, we fix a family $q:\mathcal{Y}\rightarrow S$ of deformations of complex manifolds, and study deformations of holomorphic maps into the family $q:\mathcal{Y}\rightarrow S$. We shall prove two fundamental theorems and a theorem of stability.

Finally in § 7, we shall study deformations of compositions of holomorphic maps.

Some of the results were announced in [2] and [3].

An application was reported in [3]. Details will appear in [5].

We shall employ the notation of Part I.

§ 4. Deformations of holomorphic maps (general case).

Let Y be a complex manifold. A family $(\mathcal{X}, \Phi, p, M)$ of holomorphic maps into Y consists of a family $p:\mathcal{X}\rightarrow M$ of compact complex manifolds and a holomorphic map $\Phi:\mathcal{X}\rightarrow Y\times M$ such that $pr_2\circ\Phi=p$, where pr_2 denotes the projection onto the second factor (see Definition 1.1, in § 1, Part I). Let 0 be a point on M, $X=X_0$, and let $f=\Phi_0: X\rightarrow Y$ be the holomorphic map induced by Φ. Letting Θ_X and Θ_Y denote the sheaf of germs of holomorphic vector fields on X and Y, respectively, we have a canonical homomorphism $F:\Theta_X\rightarrow f^*\Theta_Y$.

Let $\mathfrak{U}=\{U_i\}$ be a finite Stein covering of X. For any sheaf $\mathcal{F}$ on X, we let $C^q(\mathfrak{U}, \mathcal{F})$ and $\mathcal{Z}^q(\mathfrak{U}, \mathcal{F})$ denote, respectively, the group of q-cochains and the group of q-cocycles with coefficients in $\mathcal{F}$ with respect to the covering $\mathfrak{U}$. We define the coboundary map $\delta: C^q(\mathfrak{U}, \mathcal{F})\rightarrow C^{q+1}(\mathfrak{U}, \mathcal{F})$ as usual (see [1], II. 5.1, for example).