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Martin boundary over an isolated singularity of rotation free density

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Consider a *density* $P(z)dxdy$ on a Riemann surface R, i.e. a 2-form $P(z)dxdy$ whose coefficients $P(z)$ are nonnegative locally Hölder continuous functions of local parameters $z=x+iy$ on R. Let δ be an isolated parabolic ideal boundary component of R. This means that there exists the base $\{\Omega^*\}$ of neighborhoods of the point δ in the Kerékjártó-Stoilow compactification of R such that each $\Omega=\Omega^*\cap R$ is an *end* of R, i.e. a subregion of R with compact analytic relative boundary $\partial\Omega$ and a single ideal boundary component δ. The parabolicity of δ is characterized by the parabolicity of the double $\hat{\Omega}$ of $\Omega=\Omega^*\cap R$ about $\partial\Omega$ for every $\Omega^*\in\{\Omega^*\}$. To describe a potential theoretic behavior of P at δ we introduce the *P-elliptic dimension*, $\dim_P\delta$, of δ as follows: Let $\mathcal{P}_P(\Omega)$ be the half module of nonnegative solutions u of the elliptic equation

$$\Delta u(z)=P(z)u(z) \qquad (\text{i.e. } d^*du(z)=u(z)P(z)dxdy)$$

on Ω with continuously vanishing boundary values on $\partial\Omega$ for $\Omega=\Omega^*\cap R$ with $\Omega^*\in\{\Omega^*\}$. Since $\mathcal{P}_P(\Omega)$ are isomorphic to each other as half modules for all Ω (Ozawa [**15**, **16**]), the common half module structure $\mathcal{P}_P(\delta)$ is determined only by δ and P. Then we define $\dim_P\delta$ to be the dimension of $\mathcal{P}_P(\delta)$, i.e. the minimal cardinal number of sets of generators of $\mathcal{P}_P(\delta)$. The simplest δ is the δ_0 which is represented as the origin $z=0$ of the punctured disk $0<|z|<1$, i.e. there exists $\Omega^*\in\{\Omega^*\}$ such that Ω^* is represented as $|z|<1$ and $\Omega=\Omega^*\cap R$ as $0<|z|<1$. The simplest density is $P\equiv 0$, i.e. $P(z)\equiv 0$ for every z. The P-elliptic dimension of δ with $P\equiv 0$ is in particular referred to as the *harmonic dimension* of δ.

Two opposite extreme cases of the study of the dual dependance of $\dim_P\delta$ on (δ, P) are $\delta\rightarrow\dim_P\delta$ with the simplest P, i.e. $P\equiv 0$, and $P\rightarrow\dim_P\delta$ with the simplest $\delta=\delta_0$. It is known that there exists δ such that the harmonic dimension $\dim_0\delta$ of δ is either an arbitrary finite cardinal number n (Heins [**4**]), the countably infinite cardinal number $\mathfrak{a}$ (Kuramochi [**8**]), or the cardinal number of continuum $\mathfrak{c}$ (Constantinescu-Cornea [**2**]). These are examples of the study of $\delta\rightarrow\dim_P\delta$ with $P\equiv 0$. The starting point of the