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Periodicity and inequality

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§ 0. Introduction.

In this paper, we consider a certain kind of periodicity by Fubini's theorem. Next in such a point of view, we prove the following inequality.

$$\Gamma(n/2)/2\sqrt{\pi}\,\Gamma((n+1)/2) < \max_{I\subset\{1,2,\cdots,p\}} \|\sum_{i\in I} a_i\| \Big/ \sum_{j=1}^{p} \|a_j\|,$$

where a_j ($j=1, 2, \cdots, p$) is a real n-dimensional vector ($n \geqq 2$).

For example, A. Pietsch [2] used the lemma such that for any set of complex numbers $\{a_j \in \boldsymbol{C};\ j \in J\}$, where $|\sum_{i\in I} a_i| \leqq r$ for all finite subset I of J, we have $\sum_{i\in I} |a_i| \leqq 4r$ for all I.

We can take π in place of 4, and this estimate is the best, moreover this is the generalization of Blaschke's theorem on the oval.

§ 1. Periodicity and Fubini's theorem.

THEOREM 1. *Let* $H(X, Y)$ *be* (1) *real valued bounded measurable function on* $\boldsymbol{R}^n \times \boldsymbol{R}^n$, (2) $H(X, Y) = H(Y, X)$, (3) *there exist* M *and*

$$M = \lim_{T_1, T_2, \cdots, T_n \to \infty} (2^n T_1 T_2 \cdots T_n)^{-1} \int_{-T_1}^{T_1} \cdots \int_{-T_n}^{T_n} H(X, Y)dY,$$

and this convergence is uniform. Let $\boldsymbol{B}$ *be* $[0, 1]^n$, $n \geqq 1$. *Let* $C_{\boldsymbol{B}}^1$ *be the set of all once continuously differentiable functions on* $\boldsymbol{B}$. *Then*

$$\sup_{f\in C_{\boldsymbol{B}}^1} \inf_{Y\in \boldsymbol{R}^n} \int_{\boldsymbol{B}} H(f(Z), Y)dZ = M = \inf_{f\in C_{\boldsymbol{B}}^1} \sup_{Y\in \boldsymbol{R}^n} \int_{\boldsymbol{B}} H(f(Z), Y)dZ.$$

PROOF. For any $f \in C_{\boldsymbol{B}}^1$, any $T_1, T_2, \cdots, T_n > 0$, $H(f(Z), Y)$ is a bounded measurable function on $\boldsymbol{B} \times [-T_1, T_1] \times [-T_2, T_2] \times \cdots \times [-T_n, T_n]$. By Fubini's theorem, we have

$$(2^n T_1 T_2 \cdots T_n)^{-1} \int_{-T_1}^{T_1} \cdots \int_{-T_n}^{T_n} \int_{\boldsymbol{B}} H(f(Z), Y)dZdY$$

$$= \int_{\boldsymbol{B}} (2^n T_1 T_2 \cdots T_n)^{-1} \int_{-T_1}^{T_1} \cdots \int_{-T_n}^{T_n} H(f(Z), Y)dYdZ.$$