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Integral equation associated with some non-linear evolution equation

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§1. Introduction and Theorem.

In [1] G. Webb established the existence and uniqueness of a global solution of the integral equation

$$U(t)x=T(t)x-\int_{0}^{t}T(t-s)Bu(s)x\,ds\,.$$

associated with the non-linear evolution equation

$$du/dt+Au(t)+Bu(t)=0$$

in some Banach space X. Here A is a closed, densely defined, linear m-accretive operator from X to itself, $T(t)$ is the semigroup generated by $-A$, and B is a continuous, everywhere defined, non-linear accretive operator from X to itself. This result was extended by K. Maruo and N. Yamada [2] to the case where A and B are both dependent on t. In the present paper it is shown that a similar result remains valid if B is a not necessarily everywhere defined operator depending on t provided that $-A$ is the infinitesimal generator of an analytic semigroup.

Throughout this paper X will denote a Banach space with norm $\|\ \|$. We impose the following conditions on the operator A and $B(t)$, $0\leqq t\leqq T<+\infty$:

(I) A is a closed, densely defined, linear m-accretive operator from X to itself. $T(t)$ which is the semigroup generated by $-A$ is an analytic semigroup.

In what follows we assume that the origin belongs to the resolvent set of A without loss of generality.

(II) For each $t\in[0, T]$ $B(t)$ is an accretive, nonlinear operator from X to itself.

(III) There exist numbers α, α' with $\alpha>0$, $\alpha'\geqq 0$, $\alpha+\alpha'<1$ and a positive non-decreasing function $l(x)$ defined on $[0, \infty)$ such that

(i) $D(A^{\alpha})\subset D(B(t))$ for $0\leqq t\leqq T$;

(ii) for any $\varepsilon>0$ and $t\in[0, T]$ there exists a positive number δ depend-