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On homogeneous P^{N}-bundles over an abelian variety

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Let $M=M(T, \pi, P^{N})$ be a P^{N}-bundle over an abelian variety T, $G=\mathrm{Aut}^{0} M$ and $H=\mathrm{Aut}^{0} T$ the connected components of the complex Lie groups containing the identities of all holomorphic automorphisms of M and T respectively. Then there exists a holomorphic homomorphism π_{*} of G into H canonically induced by π.

M is said to be a *homogeneous bundle* if π_{*} is surjective. If M is a bundle defined by a homomorphism of the fundamental group Γ of T into $PGL(N)$, it is called a *flat bundle*.

In § 1, we shall prove the following proposition.

PROPOSITION. *Let M be a P^{N}-bundle over an abelian variety T. Then M is a homogeneous bundle if and only if it is a flat bundle.*

Let α be a homomorphism of Γ into $PGL(N)$. We call α *of finite type* if $\mathrm{Im}\,\alpha$ is a finite group. In § 2, we shall prove the following proposition.

PROPOSITION. *Let M be a flat P^{N}-bundle over an abelian variety T defined by a homomorphism α. If α is of finite type, then*

1) *$A\times P^{N}$ is a finite holomorphic covering manifold of M, where $A=C^{n}/\ker\alpha$,*

2) *there exists a Kähler metric canonically induced by that of $A\times P^{N}$ such that the corresponding Ricci curvature of M is positive semi-definite.*

A connected compact complex manifold M is called an *almost homogeneous manifold* if there exists a complex subgroup G of $\mathrm{Aut}\, M$ such that the G-orbit through some point of M contains an open subset of M.

COROLLARY. *Assume that $N+1$ is a prime number. If the bundle space of a P^{N}-bundle M over an abelian variety T is an almost homogeneous manifold, then there exists a flat vector bundle E over T such that M is the projection of E.*

We shall give an example of an almost homogeneous P^{3}-bundle over an abelian variety T which is not the projection of a flat vector bundle over T.

In § 3, we shall classify homogeneous P^{2}-bundles over an abelian variety T and give a necessary and sufficient condition that such a bundle space is an almost homogeneous manifold.