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A correction to "Theta series and automorphic forms on GL_2"

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The proof of Lemma 2 in [2] is not correct if the base field F is of characteristic 2. We give here a proof which is valid for any characteristic. Rather than proving the lemma in its original form, it is more convenient to prove it for every irreducible admissible representation (not necessarily pre-unitary).

1. We keep the notation in [2]. Let π be an irreducible admissible representation of $\mathcal{H}(\mathcal{K}_A^\times)$ in a vector space $\mathcal{V}$. By [**1**, Proposition 9.1] π is the tensor product of irreducible admissible representations π_v of $\mathcal{H}(\mathcal{K}_v^\times)$ in the space $\mathcal{V}_v$. Let $\mathfrak{d}=\otimes\mathfrak{d}_v$ be an irreducible representation of K^1, $\mathfrak{d}_v$ being an irreducible representation of K_v^1. Almost all $\mathfrak{d}_v$ are the identity representations. We define $\mathcal{V}(\mathfrak{d})$ or $\mathcal{V}_v(\mathfrak{d}_v)$ in the same way as in [**2**, § 3, No. 3].

LEMMA. *$\mathcal{V}_v(\mathfrak{d}_v)$ is finite dimensional. Moreover, if the restriction of π_v to K_v contains the identity representation and if $\mathfrak{d}_v=1$, then $\mathcal{V}_v(\mathfrak{d}_v)$ is one-dimensional.*

PROOF. If $\mathcal{K}_v$ is a division algebra, the first assertion is trivial since π_v itself is finite dimensional, and the second assertion follows from the fact that if π_v contains the identity representation of K_v, then π_v is one-dimensional.

Let us assume that $\mathcal{K}_v^\times=GL_2(F_v)$.

i) π_v is a representation of principal series (special representations are included). In this case π_v is realized in a subspace of some $\mathcal{B}(\mu_1, \mu_2)$. Since $GL_2(F_v)=TK_v^1$, a function in $\mathcal{B}(\mu_1, \mu_2)$ is determined by its restriction to K_v^1. A function φ on K_v^1 which transforms according to $\mathfrak{d}_v$ by $\varphi(g)\to\varphi(gk)$ $(k\in K_v^1)$ is a linear combination of the coefficients of $\mathfrak{d}_v$. Hence they form a finite dimensional space. This proves the first assertion. The second assertion is obvious.

ii) π_v is absolutely cuspidal (F_v is a non-archimedean local field). Let the notation be the same as in [**2**, § 5, No. 6], and assume that Ψ is a character of F_v of conductor $\mathfrak{o}_v$. Put

$$H_m=\{k\in K_v^1\,|\,k\equiv 1\ (\mathrm{mod}\ \mathfrak{p}^m)\}\,.$$

It is enough to prove that the space $\mathcal{V}_m$ of all H_m-invariant functions in