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Monodromy representations of homology of certain elliptic surfaces

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Introduction.

In this paper we shall determine global monodromy representations of certain basic elliptic surfaces over a complex projective line $\boldsymbol{P}^1(\boldsymbol{C})$. Such a surface has a following normal form (Kas [2]); Let $\boldsymbol{P}^2(\boldsymbol{C})$ be a complex projective plane with homogeneous coordinate (x, y, z). We take two copies $W_0=\boldsymbol{P}^2(\boldsymbol{C})\times \boldsymbol{C}_0$ and $W_1=\boldsymbol{P}^2(\boldsymbol{C})\times \boldsymbol{C}_1$ of the product $\boldsymbol{P}^2(\boldsymbol{C})\times \boldsymbol{C}$ and form their union

$$W^k=W_0\cup W_1 \qquad (k=1, 2, \cdots)$$

by identifying $(x, y, z, u)\in W_0$ with $(x_1, y_1, z_1, u_1)\in W_1$ if and only if

$$u^{2k}x_1=x, \quad u^{3k}y_1=y, \quad z_1=z, \quad uu_1=1\,.$$

Similarly we define

$$\Delta=\boldsymbol{C}_0\cup \boldsymbol{C}_1\,,$$

where we identify $u\in \boldsymbol{C}_0$ with $u_1\in \boldsymbol{C}_1$ if and only if $uu_1=1$. For a point $(\tau, \sigma)=(\tau_0, \tau_1, \cdots, \tau_{4k}, \sigma_1, \cdots, \sigma_{6k})$ in the space $\boldsymbol{C}^{10k+1}$, we set

$$g_{4k}(u)=\tau_0u^{4k}+\tau_1u^{4k-1}+\cdots+\tau_{4k}\,,$$

$$h_{6k}(u)=u^{6k}+\sigma_1u^{6k-1}+\cdots+\sigma_{6k}\,.$$

Then the basic elliptic surface $B_k(\tau, \sigma)$ over $\Delta=\boldsymbol{P}^1(\boldsymbol{C})$ is defined by

$$y^2z-4x^3+g_{4k}(u)xz^2+h_{6k}(u)z^3=0 \qquad \text{in } W_0\,,$$

$$y_1^2z_1-4x_1^3+u_1^{4k}g_{4k}(1/u_1)x_1z_1^2+u_1^{6k}h_{6k}(1/u_1)z_1^3=0 \qquad \text{in } W_1\,.$$

The projection Ψ of $B_k(\tau, \sigma)$ onto Δ is defined by

$$\Psi: (x, y, z, u)\longmapsto u$$

$$(x_1, y_1, z_1, u_1)\longmapsto u_1\,.$$

We simply denote by $u=\infty$ the point $u_1=0$ on Δ.

We define two polynomials $D_k(u)$ and $\tilde{D}_k(u_1)$, respectively, by