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On continuation of regular solutions of partial differential equations with constant coefficients

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Introduction.

This paper deals with the problem of continuation of real analytic solutions of partial differential equations with constant coefficients. In [3], [4] we have considered the following case: Let K and U be compact convex and open convex subsets of $\boldsymbol{R}^n$ such that $K \subset U \subset \boldsymbol{R}^n$. Let $\mathcal{A}_p$ denote the real analytic solutions of the partial differential equation $p(D)u=0$ with constant coefficients. Then the quotient space $\mathcal{A}_p(U\backslash K)/\mathcal{A}_p(U)$ does not depend on U and represents the obstruction of extensibility of real analytic solutions defined outside the exceptional set K to a neighborhood of K. A satisfactory result was given there: For the single operator p, it says that $\mathcal{A}_p(U\backslash K)/\mathcal{A}_p(U)=0$ *if and only if the characteristic polynomial* $p(\zeta)$ *has no elliptic irreducible component.* (As for systems see [4].) In this paper we consider a case somewhat generalizing the preceding one: Let H be an open half space in $\boldsymbol{R}^n$; $K_1=K\cap H$, where K is compact and convex as above; U_1 be an open convex neighborhood of K_1 in H. We discuss conditions for $\mathcal{A}_p(U_1\backslash K_1)/\mathcal{A}_p(U_1)=0$, and give some sufficient conditions (Theorems 2.6, 2.7, and 2.12). In case $K\subset H$ this problem reduces to the preceding one.

We adopt the method employed by Grušin [1], who studied the isolated singularities of infinitely differentiable solutions. Since we treat here the sets "with boundary", we need a new (*relative*) type of *Phragmén-Lindelöf theorem* (Lemma 2.9) which plays an essential role in our method.

In § 1 we consider the same problem for hyperfunction solutions and obtain a necessary and sufficient condition for the extensibility. The obtained result is used in the proof of theorem 2.6 for real analytic solutions. Though we can consider similar problems for other classes of regular solutions, we mainly concern ourselves with real analytic solutions of single operators here. Some of the remaining cases will be treated in future.

A part of these results was announced in [6]. Some of them has been

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