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Almost polar sets and an ergodic theorem

By Masatoshi FUKUSHIMA

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§ 1. Introduction.

We will develop a potential theory for two Markov processes which are in duality and apply it to an extension of the Chacon-Ornstein ergodic theorem.

Let X be a locally compact separable Hausdorff space and m be a positive Radon measure on X. Consider standard Markov processes $\boldsymbol{M}=(X_t, P_x)$ and $\hat{\boldsymbol{M}}=(\hat{X}_t, \hat{P}_x)$ which are in duality with respect to m in the sense that the equality

$$(1.1) \qquad (f, T_t g)=(\hat{T}_t f, g), \qquad t>0,$$

holds for any non-negative Borel functions f and g on X. Here T_t (resp. $\hat{T}_t$) is the semi-group associated with $\boldsymbol{M}$ (resp. $\hat{\boldsymbol{M}}$) and (f, g) is the integral $\int_X f(x)g(x)m(dx)$. The quantities relative to the dual process $\hat{\boldsymbol{M}}$ are denoted with ^ and designated by the prefix co-. Notice that the present duality is much weaker than that of Blumenthal-Getoor [2; VI] and we do not assume the absolute continuity of resolvents or transition probabilities.

A set $A\subset X$ is said to be *almost polar* if there is a Borel set B such that $A\subset B$ and

$$(1.2) \qquad P_x(\sigma_B<+\infty)=0 \qquad \text{for } m\text{-a.e. } x\in X,$$

where σ_B is the hitting time $\inf\{t>0;\ X_t\in B\}$. "Quasi-everywhere" or "q. e." will mean "except on an almost polar set".

Recently the notion of almost polarity was employed independently by S. Port and C. Stone [12] for additive processes with m being the Haar measure and by the author [8] for general m-symmetric Markov processes whose associated Dirichlet spaces are regular[1]. In both cases, almost polar sets were identified with the sets of λ-capacity zero, the λ-capacity being defined suitably according to the respective situations. When $\boldsymbol{M}$ is the Brownian motion on R^3, the almost polar set is just the set of the Newtonian outer capacity zero.

1) Almost polar sets are called "essentially polar" in [12] and "polar" in [8].