J. Math. Soc. Japan
Vol. 25, No. 4, 1973

Cohomologies over commutative Hopf algebras

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with **Appendix**

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(Received Jan. 30, 1973)

In [2], Sweedler has investigated a cohomology theory for module algebras over a given cocommutative Hopf algebra.

The purpose of this paper is to discuss some dual theories of [2]. In section 2, we give the definitions of cohomology groups for comodules, comodule coalgebras, comodule Hopf algebras and comodule algebras over a given commutative Hopf algebra. A familiar example of commutative Hopf algebras is the coordinate ring of an affine algebraic group. Section 3 deals with relations between these cohomology groups. Sections 4 and 5 contain the extension theory of coalgebras and Hopf algebras, which is the precisely dual statements of [2]. In section 6, we compute the cohomology groups for a special comodule algebra. Finally, section 7 gives a result on the conjugacy of the coradical splittings of commutative Hopf algebras over a field of characteristic 0.

§ 1. Preliminaries.

All vector spaces are over the ground field k. Our notation and terminology are essentially those used in [3]. One difference; if C is a coalgebra and $\phi: V\to C\otimes V$ is the structure map of a (left) C-comodule V, we sometimes write $\phi(v)=\sum v_{(C)}\otimes v_{(V)}$ for all $v\in V$.

1.1. DEFINITIONS. Let H be a Hopf algebra. The unit map $u_H: k\to H \cong H\otimes k$ gives k the structure of a left H-comodule. An algebra D which is a left H-comodule is called a left *H-comodule algebra* if $M_D: D\otimes D\to D$ and $u_D: k\to D$ are H-comodule maps. ($D\otimes D$ has the natural H-comodule structure.)

A coalgebra B which is a left H-comodule is called a left *H-comodule coalgebra* if $\Delta_B: B\to B\otimes B$ and $\varepsilon_B: B\to k$ are H-comodule maps.

A Hopf algebra L which is a left H-comodule is called a left *H-comodule Hopf algebra* (or *H-Hopf action* on L) if M_H, u_H, Δ_H and ε_H are H-comodule maps.