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On exceptional values of meromorphic functions of divergence class

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§ 1. Introduction.

In this paper, following the methods used in [5] and [6], we investigate the value distribution of meromorphic functions of divergence class or of infinite order in the plane $|z|<\infty$. Let $f(z)$ be a meromorphic function having divergence class of order ρ, $0<\rho<\infty$, in $|z|<\infty$; that is, $\int^{\infty}T(t,f)/t^{1+\rho}dt=\infty$. Then, it is known that there are at most two G-exceptional values which satisfy $\int^{\infty}N(t,a)/t^{1+\rho}dt=O(1)$. Further, these values are not always exceptional in the sense of Nevanlinna ([7]) and conversely there is a meromorphic function $g(z)$ of divergence class such that $\delta(0,g)=1$ and the value 0 is not G-exceptional (Example 2, § 4). These examples show that these two notions of exceptionality of values are independent of each other in a sense. Then, how many values are there for $f(z)$ which satisfy $\delta(a,f)=1$ or are G-exceptional? We start from this question, discuss some relations among Borel exceptional values, G-exceptional values and Nevanlinna exceptional values, and introduce a new notion of exceptionality of values for meromorphic functions of divergence class (§ 2).

Any meromorphic function $h(z)$ of infinite order in $|z|<\infty$ is of divergence class in a sense, because for any large number λ, $\int^{\infty}T(t,h)/t^{1+\lambda}dt=\infty$. Analogizing with the case of finite order, we introduce notions of exceptionality of values for meromorphic functions of infinite order and give some relations with the Nevanlinna deficient values (§ 3).

Some examples are given in § 4.

We will use the symbols of the Nevanlinna theory:

$$T(r,f),\quad m(r,a),\quad N(r,a),\quad \delta(a,f),\quad S(r,f)\quad \text{etc.}$$

freely ([2], [4]).

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