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Linear evolution equations of "hyperbolic" type, II

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§ 0. Introduction.

In a previous paper [1] with the same title, we studied linear evolution equations of the form

$$\text{(E)} \qquad du/dt+A(t)u=f(t), \qquad 0 \leqq t \leqq T, \qquad u(0)=\phi,$$

in a Banach space X. The main purpose of the present paper is to prove some approximation theorems related to (E). Before doing so, however, we find it useful to strengthen some of the fundamental results of [1] by replacing the strong continuity of certain operator-valued functions with strong measurability.

As regards the approximation theorems, we considered similar problems in another paper [2] under somewhat different assumptions. However, only approximation in the X-norm was considered in [2]. In what follows we are primarily interested in approximation in a stronger norm.

The results of this paper are useful, among others, in applications to nonlinear evolution equations. Such applications will be discussed in other publications.

The paper is self-contained in definitions and statements of theorems, but their proof heavily leans on [1].

§ 1. Quasi-stability.

Let X be a Banach space. We denote by $G(X)$ the set of all negative generators of C_0-semigroups on X. A family $A=\{A(t)\}$ of elements of $G(X)$, defined for a.e. $t \in I=[0, T]$, is said to be *quasi-stable* if

$$\text{(1.1)} \qquad \| \prod_{j=1}^{k} (A(t_j)+\lambda_j)^{-1} \| \leqq M \prod_{j=1}^{k} (\lambda_j-\beta(t_j))^{-1}$$

for every finite family of real numbers $\{t_j, \lambda_j\}$ such that $0 \leqq t_1 \leqq \cdots \leqq t_k \leqq T$, $\lambda_1 > \beta(t_1), \cdots, \lambda_k > \beta(t_k)$, where M is a constant and β is a real-valued, upper-integrable function (in the Lebesgue sense) defined a.e. on I. Of course only

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