

## On the Hecke operators for $\Gamma_0(N)$ and class fields over quadratic number fields

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### Introduction.

It was shown by Shimura [3, § 7.7], [5] that the eigen-values of Hecke operators for the cusp forms of “Neben”-type (in Hecke’s sense) are closely connected with the reciprocity law in certain abelian extensions of a real quadratic field, and such extensions can be generated by the coordinates of certain points of finite order on an abelian variety associated with the cusp forms. Especially, in [5], *some fundamental theorems* about a class-field-theoretical treatment of these extensions in the case of arbitrary levels, and various detailed examples in the case of square-free levels were given. As a continuation of this theory, we are naturally led to investigate the eigen-values of Hecke operators for the cusp forms of an arbitrary level, especially, the case in which the level is divisible by a prime power  $p^n$  ( $n > 1$ ). Recently, H. Hijikata [1] has succeeded in extending the result of Eichler (the trace formula for Hecke operators) to arbitrary levels including both “Haupt” and “Neben”-types, and moreover, applying this Hijikata’s formula, in [7], one of the authors of the present note has given an explicit trace formula for a certain restricted part of the space of cusp forms of “Haupt”-type for arbitrary levels. By means of these formulae, we can obtain some numerical eigen-values of Hecke operators for the “essential part” (see T. Miyake [2] and Shimura [5, p. 133]) of the spaces. Though Shimura [5] considered only the cases of “Neben”-type, looking at [3, Prop. 3.64] and [5, § 9] carefully, we can also expect to develop the idea of [5] in the case of “Haupt”-type if levels are divisible by a higher power of a prime. Actually, Shimura [6] indicates this possibility by giving *a twisting operator* and an abelian variety associated to the cusp forms of weight 2 of “Haupt”-type analogous to “Neben”-type (see text or [6]). Now one of the aims of the present note (§ 1 and § 2 below) is to investigate this abelian variety. More precisely, take a cusp form  $f(z)$  (which is a common eigen-function of Hecke operators) of weight 2 with respect to  $\Gamma_0(p^n)$ ,  $n > 1$  of “Haupt”-type. By applying the result of [6] to the group