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On restriction algebras of tensor algebras

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§ 1. The main results.

Let X be a compact (non-empty, Hausdorff) space, and $C(X)$ (resp. $D(X)$) the Banach algebra of all continuous (resp. bounded) complex-valued functions on X. Let Y be another compact space, and consider the Banach algebras

$$V(X, Y) = C(X) \widehat{\otimes} C(Y), \quad \text{and} \quad V_D(X, Y) = D(X) \widehat{\otimes} D(Y),$$

both being endowed with the projective tensor product norm (see [**13**; Chap. 1 and 2]). Then we have the natural imbeddings

$$V(X, Y) \subset V_D(X, Y) \subset D(X \times Y),$$

where the first one is an isometric homomorphism and the second one is a norm-decreasing one-to-one homomorphism (see Theorems 4.1 and 4.3 in [7]). For an arbitrary closed subset E of the product space $X \times Y$, we define the Banach algebras $V(E)$ and $\tilde{V}(E)$ as in [**14**]. Similarly, we define the algebra $V_D(E)$ as follows. The space $V_D(E)$ is the subalgebra of $D(E)$ consisting of all functions $f \in D(E)$ that have an expansion of the form

$$f(x, y) = \sum_{n=1}^{\infty} g_n(x) h_n(y) \qquad ((x, y) \in E),$$

where $g_n \in D(X)$, $h_n \in D(Y)$ and

$$M = \sum_{n=1}^{\infty} \|g_n\|_{D(X)} \cdot \|h_n\|_{D(Y)} < \infty;$$

the norm $\|f\|_{V_D(E)}$ is defined to be the infimum of the M's taken over all expansions of f in the above form. Thus we have

$$V(E) \subset \tilde{V}(E) \subset C(E) \quad \text{and} \quad V(E) \subset V_D(E) \subset D(E).$$

It is easy to see that these four imbeddings are all norm-decreasing. We also write $V_C(E) = V_D(E) \cap C(E)$, which is clearly a closed subalgebra of $V_D(E)$.

Let now E be an arbitrary subset of the product space $X \times Y$. It is called *rectangular* if $E = \pi_X(E) \times \pi_Y(E)$, where π_X and π_Y denote the canonical pro-