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On deformations of holomorphic maps I

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Notation.

$\boldsymbol{C}$: the field of complex numbers.
X, Y, Z: compact complex manifolds.
$\mathcal{X}$, $\mathcal{Y}$, $\mathcal{Z}$, M, M', N: (connected) complex manifolds.
If $f: X \to Y$ is a holomorphic map,
$\Theta_{X/Y}$: the sheaf of germs of relative vector fields,
$\Theta_X = \Theta_{X/C}$: the sheaf of germs of holomorphic vector fields on X.
If E is a vector bundle (or a locally free sheaf) on X,
$\mathcal{A}^{0,q}(E)$: the sheaf of germs of differentiable $(0, q)$-forms with coefficients in E,
$A^{0,q}(E) = \Gamma(X, \mathcal{A}^{0,q}(E))$.
If $p: \mathcal{X} \to M$ is a family of compact complex manifolds,
X_t: the fibre over $t \in M$.
If $q: \mathcal{Y} \to N$ is another family of compact complex manifolds and if
$(\Phi, s): (\mathcal{X}, p, M) \to (\mathcal{Y}, q, N)$ is a morphism of families (i.e., $\Phi: \mathcal{X} \to \mathcal{Y}$, $s: M \to N$, $q \circ \Phi = s \circ p$),
$\Phi_t: X_t \to Y_{s(t)}$: the holomorphic map induced by Φ.
If $\{U_i\}$ is an open covering of X
$U_{ij\cdots k} = U_i \cap U_j \cap \cdots \cap U_k$.
For any vector $t = (t_1, t_2, \cdots, t_r)$,
$|t| = \max_{\lambda} |t_\lambda|$.
We denote by ν the multi-index $(\nu_1, \cdots, \nu_r)$, and
$t^\nu = t_1^{\nu_1} t_2^{\nu_2} \cdots t_r^{\nu_r}$,
$|\nu| = \nu_1 + \nu_2 + \cdots + \nu_r$.

Introduction.

The modern deformation theory has begun with the splendid work of Kodaira-Spencer [5] followed by [6], [7]. Moreover Kodaira has investigated families of submanifolds of a fixed complex manifold in [8]. The next natural problem is to investigate "deformations of holomorphic maps." First