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An addition formula for Kodaira dimensions of analytic fibre bundles whose fibres are Moišezon manifolds

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§ 0. Introduction.

Let K_M be the canonical line bundle of a compact complex manifold M. If $\dim H^0(M, \mathcal{O}(K_M^{\otimes m}))=N+1\geqq 2$ we have a meromorphic mapping $\Phi_{mK}: M\rightarrow \boldsymbol{P}^N$ of M into $\boldsymbol{P}^N$. When m is a positive integer the meromorphic mapping Φ_{mK} is called pluricanonical mapping. In this case the *Kodaira dimension* $\kappa(M)$ of M is, by definition

$$\kappa(M)=\max_{m\in L}\dim \Phi_{mK}(M)\ ,$$

where $L=\{m\in \boldsymbol{N} \mid \dim H^0(M, \mathcal{O}(K_M^{\otimes m}))\geqq 2\}$. When $H^0(M, \mathcal{O}(K_M^{\otimes m}))=0$ for all positive integers, we define the Kodaira dimension $\kappa(M)$ of M to be $-\infty$. When $\dim H^0(M, \mathcal{O}(K_M^{\otimes m}))\leqq 1$ for all positive integers m and there exists a positive integer m_0 such that $\dim H^0(M, \mathcal{O}(K_M^{\otimes m_0}))=1$, we define $\kappa(M)=0$. As for the fundamental properties of Kodaira dimension, see [**3**].

By a Moišezon manifold V we mean an n-dimensional compact complex manifold that has n algebraically independent meromorphic functions.

The main purpose of the present paper is to prove the following

MAIN THEOREM. *Let $\pi: M\rightarrow S$ be a fibre bundle over a compact complex manifold S whose fibre and structure group are a Moišezon manifold V and the group* Aut(V) *of analytic automorphisms of V respectively. Then we have an equality*

$$\kappa(M)=\kappa(V)+\kappa(S)\ .$$

To prove Main Theorem we need to analyze the action of $\mathrm{Aut}(V)$ on the vector space $H^0(V, \mathcal{O}(K_V^{\otimes m}))$. More generally the group $\mathrm{Bim}(V)$ of all bimeromorphic mappings of V acts on $H^0(V, \mathcal{O}(K_V^{\otimes m}))$ for any positive integer m. Hence we have a representation $\rho_m: \mathrm{Bim}(V)\rightarrow GL(H^0(V, \mathcal{O}(K_V^{\otimes m})))$. We call this representation pluricanonical representation. A group G is called periodic if each element g of G is of finite order. In § 1 we shall prove the following

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