J. Math. Soc. Japan
Vol. 25, No. 3, 1973

Uniformly hyperfinite algebras and locally compact transformation groups

By Yukimasa OKA

(Received March 10, 1971)
(Revised Oct. 7, 1972)

§**0.** J. Glimm [4] and E. G. Effros-F. Hahn [2] introduced and studied the C^*-algebras associated with locally compact transformation groups. In this paper, we shall represent uniformly hyperfinite algebras and the C^*-algebras of completely continuous operators as transformation group C^*-algebras.

Let $G^{(k)}$ $(k=1, 2, \cdots)$ be cyclic groups of finite order q_k, and consider the product groups $G_n=\prod_{k=1}^{n} G^{(k)}$ $(n=1, 2, \cdots)$. By left multiplications, $G^{(k)}$ and G_n are transformation groups on themselves. We denote by $Z^{(k)}$ and Z_n the groups $G^{(k)}$ and G_n which are considered as the underlying spaces of transformation groups. Then $(G^{(k)}, Z^{(k)})$ and (G_n, Z_n) are discrete finite transformation groups. Let $Z=\prod_{k=1}^{\infty} Z^{(k)}$ and let G be the restricted direct product $\prod_{k=1}^{\infty} G^{(k)}$ of $G^{(k)}$. Then (G, Z) is a locally compact transformation group such that G is a discrete countable group and Z is a compact Hausdorff space. Let $\mathfrak{A}(G^{(k)}, Z^{(k)})$ $(k=1, 2, \cdots)$, $\mathfrak{A}(G_n, Z_n)$ $(n=1, 2, \cdots)$, and $\mathfrak{A}(G, Z)$ be transformation group C^*-algebras associated with the transformation groups $(G^{(k)}, Z^{(k)})$ $(k=1, 2, \cdots)$, (G_n, Z_n) $(n=1, 2, \cdots)$, and (G, Z), respectively ([2], [4]). Then we shall show that $\mathfrak{A}(G, Z)$ is a uniformly hyperfinite algebra of type $\{p_n\}$, where $p_n=q_1q_2\cdots q_n$ $(n=1, 2, \cdots)$, and $\mathfrak{A}(G, Z)=C^*\text{-}\lim_n \mathfrak{A}(G_n, Z_n)=\otimes_k \mathfrak{A}(G^{(k)}, Z^{(k)})$ (Theorem 5). Conversely, let $\mathfrak{A}$ be a uniformly hyperfinite algebra of type $\{p_n\}$. Then we shall show that there exists a transformation group (G, Z) such that $\mathfrak{A}$ is $*$-isomorphic to $\mathfrak{A}(G, Z)$ where (G, Z) is as above (Corollary 6).

In § 3, we shall show that if G is an infinite cyclic group, the C^*-algebra $\mathfrak{A}(G, G)$ associated with the transformation group (G, G) is $*$-isomorphic to the C^*-algebra $\mathfrak{LC}(H)$ of all completely continuous operators in a separable Hilbert space H (Theorem 9).