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Finite groups with central Sylow 2-intersections

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§ 1. Introduction.

The purpose of this paper is to clarify the structure of finite groups which satisfy the following condition:

(CI): The intersection of any two distinct Sylow 2-groups is contained in the center of a Sylow 2-group.

From now on, we call a finite group a (CI)-*group* if it satisfies (CI). The main result is the following:

THEOREM 1. *Let G be a* (CI)-*group. Then one of the following statements holds.*

(1) *G is a solvable group of* 2-*length* 1.

(2) *A Sylow* 2-*group of G is Abelian.*

(3) *G has a normal series $1 \leqq N < M \leqq G$ where N and G/M have odd order and M/N is the central product of an Abelian* 2-*group and a group isomorphic to $SL(2, 5)$.*

(4) *G contains a normal subgroup M of odd index in G which satisfies one of the following conditions:*

(4.1) *M is the direct product of an Abelian* 2-*group and a group isomorphic to $Sz(q)$, $PSU(3, q)$ or $SU(3, q)$, q a* 2-*power* > 2.

(4.2) *M is the central product of an Abelian* 2-*group and a non-trivial perfect central extension of $Sz(8)$.*

If we combine Theorem 1 with the theorems of Walter [**13**] and Bender [**2**], we obtain the following result.

THEOREM 2. *A non-Abelian simple* (CI)-*group is isomorphic to one of the following groups:*

$PSL(2, q)$, $q \equiv 0, 3, 5 \pmod 8$,

JR,

$Sz(q)$ *or*

$PSU(3, q)$, *q a power of* 2.

Here JR denotes the simple groups with Abelian Sylow 2-groups in which the centralizer of an involution t is a maximal subgroup and isomorphic to $\langle t\rangle \times E$ where $PSL(2, q) \leqq E \leqq P\Gamma L(2, q)$ with odd $q > 3$. This definition is due