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Bordism groups of dihedral groups

By Masayoshi KAMATA and Haruo MINAMI

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Let G be a finite group. By a G-manifold we mean a closed oriented manifold together with an orientation preserving action of G without fixed points or a closed weakly complex manifold together with a weakly complex structure preserving action of G without fixed points. We denote a G-manifold by a pair (M, f) where M is a G-manifold and f a free action of G on $M: G\times M\rightarrow M$ and its bordism class by $[M, f]$. Moreover we denote by $\tilde{\Omega}_{m}^{SO}(G)$ the oriented reduced bordism group of G of dimension m and by $\tilde{\Omega}_{m}^{U}(G)$ the weakly complex reduced bordism group of G of dimension m.

Let D_{n} be the dihedral group of order $2n$. In this paper the authors show a mapping splitting theorem for $\tilde{\Omega}_{m}^{SO}(D_{n})$ and $\tilde{\Omega}_{m}^{U}(D_{n})$ when n is odd and determine the additive structure of $\tilde{\Omega}_{m}^{U}(D_{p})$, p an odd prime.

In the following sections we denote $\tilde{\Omega}_{m}^{SO}(G)$ or $\tilde{\Omega}_{m}^{U}(G)$ by $\tilde{\Omega}_{m}^{L}(G)$.

§ 1. A mapping splitting theorem for $\tilde{\Omega}_{m}^{L}(G)$.

Let G be a finite group and BG a classifying space of G. Let (M, f) be a G-manifold of dimension m. Then $\pi: M\rightarrow M/G$ is a principal G-bundle and there exists a classifying map $g: M/G\rightarrow BG$. The correspondence $[M, f]\mapsto[M/G, g]$ is well-defined homomorphism of $\tilde{\Omega}_{m}^{L}(G)$ into $\tilde{\Omega}_{m}^{L}(BG)$ and we have the following known result.

THEOREM 1.1 (Conner-Floyd [1]). *The above defined homomorphism* ρ_{*}: $\tilde{\Omega}_{*}^{L}(G)\rightarrow\tilde{\Omega}_{*}^{L}(BG)$ *is an isomorphism of degree* 0 *as an* Ω_{*}^{L}*-module homomorphism.*

Let $\alpha: H\rightarrow G$ be a homomorphism of finite groups and $B\alpha: BH\rightarrow BG$ a map induced by α. We denote by $\alpha_{*}: \tilde{\Omega}_{*}^{L}(BH)\rightarrow\tilde{\Omega}_{*}^{L}(BG)$ the homomorphism induced by $B\alpha$ and we also denote $\rho_{*}^{-1}\alpha_{*}\rho_{*}: \tilde{\Omega}_{*}^{L}(H)\rightarrow\tilde{\Omega}_{*}^{L}(G)$ by α_{*}. Then we have

$$\alpha_{*}([M, f])=[G\times_{H}M, f_{G}], \qquad [M, f]\in\tilde{\Omega}_{*}^{L}(H) \tag{1.1}$$

where $G\times_{H}M=G\times M/(g, x)\sim(g\alpha(h)^{-1}, f(h, x))$, $g\in G$, $h\in H$ *and* $x\in M$ *on which* G *acts by the rule*

$$f_{G}(g, g'\times_{H}x)=gg'\times_{H}x, \qquad g, g'\in G,\ x\in M.$$