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On pseudoconvexity of complex abelian Lie groups

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§ 1. Introduction.

The purpose of this paper is to prove the following theorem.

THEOREM. *Let G be a complex abelian Lie group of complex dimension n and K the maximal compact subgroup of the connected component of G with Lie algebra $\mathfrak{k}$. Let q be the complex dimension of $\mathfrak{k}\cap\sqrt{-1}\mathfrak{k}$. Then there exists a real-valued C^{∞} function φ on G satisfying the following conditions:*

(1) *The Levi form of φ:*

$$L(\varphi, x)=\sum_{i,j=1}^{n}\frac{\partial^{2}\varphi}{\partial z_{i}\partial\bar{z}_{j}}dz_{i}d\bar{z}_{j}$$

is positive semi-definite and has $n-q$ positive eigenvalues at every point x of G, where $(z_1, z_2, \cdots, z_n)$ denotes a system of coordinates in some neighborhood of x.

(2) *The set*

$$G_{c}=\{g\in G:\ \varphi(g)<c\}$$

is a relatively compact subset of G for any $c\in\boldsymbol{R}$.

By the above theorem any complex abelian Lie group is always pseudoconvex. In the last part we shall find a complex Lie group of arbitrary dimension, on which every holomorphic function is a constant and which is pseudoconvex and 1-complete.

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§ 2. Proof of Theorem.

Since all connected components of G are biholomorphically isomorphic, we may assume that G is connected. Let $\mathfrak{O}$ be the sheaf of all germs of holomorphic functions on G. We put

$$G^{0}=\{g\in G:\ f(g)=f(e)\quad \text{for all } f\in H^{0}(G, \mathfrak{O})\}$$

where e is the unit element of G. Then Morimoto [5] proved that G^{0} is a complex abelian Lie subgroup of G and that every holomorphic function on