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On universal embeddings in matrix rings

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Given a ring R and an integer $n \geqq 1$ we have shown in [1] that there exists a commutative ring S and a homomorphism $\rho: R \rightarrow M_n(S)$, the matrix rings over S, such that every other homomorphism $\sigma: R \rightarrow M_n(K)$ with K commutative, is induced by a homomorphism $\eta: S \rightarrow K$. The category of rings considered in [1] was the category of rings not necessarily containing a unit, but S was taken to be a ring with a unit. It seems that that was not the natural assumption on S and in fact this caused an incomplete proof in the last section of [1]. In the present note, a different ring S_0 is obtained with the aid of the preceding ring S which is the natural universal object in the category of rings (not necessarily containing a unit). This is applied to give a characterization of ring whose irreducible representations are of $\dim \geqq n$. Finally, the results obtained yield a universal splitting ring of central separable algebras. A byresult is a criterion for a set of matrices to generate the full matrix ring.

§ 1. Notations and remarks.

These will follow the notations of [1]: k will denote a fixed commutative ring with a unit. All rings considered here will be k-algebras and all homomorphism will be k-homomorphism.

$M_n(R)$ will denote the $n \times n$ matrix ring over a ring R, and $M_n(\eta): M_n(R) \rightarrow M_n(S)$ will denote the homomorphism of the matrix rings induced by a homomorphism $\eta: R \rightarrow S$.

$k[x]$ will denote the free ring generated over k by a set $\{x_i\}$ of non-commutative indeterminates. Denote by $S_i = (\xi^i_{\alpha\beta})$ $\alpha, \beta = 1, 2, \cdots, n$ generic matrices of order n over k, where the $\xi^i_{\alpha\beta}$ are commutative indeterminates over k. $\Delta = k[\xi]$ will denote the ring of commutative polynomials in the ξ's; and $k[X]$ will be the subring of $M_n(\Delta)$ generated by the generic matrices.

The notation $\Delta^0 = k^0[\xi]$, $k^0[x]$, $k^0[X]$ will stand for the corresponding rings of polynomial with zero as the free coefficient.