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Semi-groups of operators in locally convex spaces

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In this paper we shall deal with the generation of not necessarily equicontinuous semi-groups of operators in locally convex spaces in a simple way which is different from that of T. Kōmura [2].

The theory of semi-groups has made progress after the fundamental works of E. Hille and K. Yosida. H. Komatsu [1], L. Schwartz [3], K. Yosida [5] and others extended the theory of semi-groups of operators in Banach spaces to locally convex spaces. They discussed it under the condition that semi-groups are equicontinuous. Their arguments are based on the following fact which plays an essential role in their theory.

For an equicontinuous semi-group $\{T_t;\ t\geqq 0\}$ *in a locally convex space* E, *the Laplace transform of* T_t *exists, and it is connected with its infinitesimal generator* A *in the following way:*

$$\int_0^\infty e^{-\lambda t}T_t x\,dt=(\lambda-A)^{-1}x\,, \tag{0.1}$$

for any $x\in E$ *and* $\operatorname{Re}\lambda>0$.

Without assumption of equicontinuity of a semi-group T_t, neither the Laplace transform of T_t nor the resolvents of its infinitesimal generator A ever exist.

In T. Kōmura [2], she dealt with semi-groups which are not necessarily equicontinuous but locally equicontinuous. To avoid the difficulty that the relation (0.1) does not necessarily hold, she introduced the notion of generalized resolvents. Generalized resolvents play an important role in her theory. To define generalized resolvents and to get their properties she used the theories of vector valued distributions and linear topological spaces attached to them and their related properties. Hence it seems for the author that the notion of generalized resolvents is not simple.

In the following of this paper we discuss the generation of semi-groups without the notion of generalized resolvents. Instead of it, we introduce the notion of asymptotic resolvents to complete our theory. Roughly speaking, asymptotic resolvents are almost resolvents, or they are parametrix of $(\lambda-A)$, where A is the infinitesimal generator of a semi-group T_t, modulo