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Scattering theory for differential operators, II, self-adjoint elliptic operators

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The present paper is a direct continuation of Part I (reference [1] which, hereafter, will be referred to as (I)) and is concerned with the application of the results obtained in (I) to the spectral and the scattering problems for the self-adjoint elliptic differential operator

$$Hu=\sum_{|\alpha|,|\beta|\leqq m} D^{\alpha}(a_{\alpha\beta}^{(1)}+a_{\alpha\beta}(x))D^{\beta}u$$

in R^{n}. Throughout the paper the same notations as in (I) will be used. Theorems etc. given in (I) will be quoted as Theorem I.2.9 for theorems, as (I.3.7) for formulas, as [I.1] for references, etc.

Recently, S. Agmon investigated the self-adjoint elliptic operator $\sum_{|\alpha|\leqq 2m} a_{\alpha}(x)D^{\alpha}u$ by the method of limiting absorption (or by a weighted elliptic estimate) and announced the results in [I.1] and a lecture quoted in (I). The result given in the present paper considerably overlaps with Agmon's results. In particular, Theorem 1.5 given below is essentially equivalent to what is announced in [I.1]. The approach to the proof, however, is different. Our work has been carried out independently of Agmon's except for the last stage where the proof of ii) of Theorem 1.5 (or Theorem I.5.21) was completed after having been stimulated by Agmon's work.

We will treat the problem as an example to which the abstract method given in (I) can be applicable.[1] The crucial tool which makes this application possible is the trace theorem in the Sobolev spaces.

§1. Assumptions and main results.

Throughout the present paper we write $D_{j}=-i\partial/\partial x_{j}$, $x=(x_{1}, \cdots, x_{n})\in R^{n}$, and $D^{\alpha}=D_{1}^{\alpha_{1}}\cdots D_{n}^{\alpha_{n}}$, where $\alpha=(\alpha_{1}, \cdots, \alpha_{n})$ is a multi-index and the differentia-

1) As is mentioned in §1 of (I), M. Š. Birman also investigated the scattering theory for general differential operators by applying his abstract method. See, e.g., [I.2].