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Approximations of nonlinear evolution equations

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§ 1. Introduction.

In this paper we are concerned with approximation of the solution to the Cauchy initial value problem

$$0 \in u'(t) + A(t)u(t)\,, \qquad u(0) = x\,. \tag{1.1}$$

The basic tool of this investigation is a theorem by Crandall and Liggett [4] which provides conditions sufficient for the existence of the infinite product, $u(t) = \lim_{n \to \infty} \prod_{i=1}^{n} (I + (t/n)A(it/n))^{-1}x$. A product of the foregoing type is often referred to as a product integral. It is not always possible to obtain a solution to (1.1), however, it may be possible to associate a product integral with a Cauchy problem. As we shall see, solutions to given Cauchy problems may often be represented by product integrals. Our main result concerns the convergence of product integrals associated with a class of approximate Cauchy problems. Several authors have studied questions of this nature (e. g., see Oharu [15], Miyadera [13], [14], [15], Brezis and Pazy [1], [2], [3] and Mermin [10], [11], and Crandall and Pazy [5]).

The author is grateful for the opportunity to see preprints of the forementioned manuscripts of Brezis and Pazy, and Crandall and Liggett. Appreciation is due G. F. Webb for suggesting the problems considered and for his invaluable criticisms. The author also wishes to thank the referee whose suggestions strengthened Theorems 2.11 and 3.1.

§ 2. Preliminaries.

Throughout this paper X will be a real Banach space. It is often useful to consider "multivalued" operators. Cauchy problems associated with these operators assume the form $0 \in u'(t) + A(t)u(t)$. We shall refer to "multivalued" operators as subsets of $X \times X$. The term operator will be exclusively reserved for operators in the usual sense.

If S is a set, let $|S| = \inf \{\|x\| \mid x \in S\}$. A subset of $X \times X$ is said to be accretive if for each $\lambda \geqq 0$ and $[x_i, y_i] \in A$, $i = 1, 2$, we have $\|(x_1 + \lambda y_1) -$