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Hypersurface of an even-dimensional sphere satisfying a certain commutative condition

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Introduction.

Let $\bar{M}$ be a Riemannian manifold which admits a linear transformation $\bar{f}$ of its tangent bundle $T(\bar{M})$. Then, the tangent bundle $T(M)$ of a hypersurface M of $\bar{M}$ naturally admits a linear transformation f induced from that of the tangent bundle of the ambient space.

On the other hand, on any hypersurface of a Riemannian manifold there is the linear transformation H of its tangent bundle which is defined by the second fundamental tensor.

It seems to be an interesting problem to consider relations between some linear algebraic conditions of these two transformations and properties of the hypersurface.

In this direction one of the authors started to study the case where the ambient space is an even-dimensional Euclidean space [4] and thereafter Yamaguchi [7], Yano [3] and the present authors [3, 5] studied the case where the ambient manifold is an odd-dimensional sphere and these transformations are commutative or anti-commutative.

However, until recently there was no known linear transformation of the tangent bundle of an even-dimensional sphere except in dimensions 2 and 6. Recently two of the present authors and Yano [1, 2] found a linear transformation of the tangent bundle of certain even-dimensional manifolds including an even-dimensional sphere.

In this paper, using this linear transformation, we study a hypersurface of an even-dimensional sphere for which this transformation commutes with the transformation H.

§ 1. Hypersurface of an even-dimensional sphere.

Let S^{2n} be an even-dimensional sphere of radius 1. Then on S^{2n} there exist a (1, 1)-tensor field $\bar{f}$, two vector fields $\bar{U}$, $\bar{V}$, two 1-forms u, v and a