J. Math. Soc. Japan
Vol. 25, No. 2, 1973

Primitive extensions of rank 4 of multiply transitive permutation groups
(Part I. The case where all the orbits are self-paired)

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(Received Dec. 6, 1971)

Introduction.

In [1] the author has determined the permutation groups which are primitive extensions of rank 3 of 4-ply transitive permutation groups. This note is a continuation of [1], and here we consider primitive extensions of rank 4 of multiply (5-ply) transitive permutation groups. Here we say that a permutation group $(\mathfrak{G}, \Omega)$ is a primitive extension of rank r of a (transitive) permutation group (G, Δ) if the following conditions are satisfied: (i) $\mathfrak{G}$ is primitive and of rank r on the set Ω, and (ii) there exists an orbit $\Delta(a)$ of the stabilizer $\mathfrak{G}_a$ $(a \in \Omega)$ such that the action of $\mathfrak{G}_a$ on $\Delta(a)$ is faithful and that $(\mathfrak{G}_a, \Delta(a))$ and (G, Δ) are isomorphic as permutation groups.

In this note we will prove the following theorem:

THEOREM 1. *Let (G, Δ) be a 5-ply transitive permutation group. If (G, Δ) has a primitive extension of rank 4 $(\mathfrak{G}, \Omega)$ such that the orbits of $\mathfrak{G}_a$ $(a \in \Omega)$ on Ω are all self-paired, then* (i) $|\Delta|=7$ *and* $G=S_7$ *or* A_7 *(symmetric and alternating groups on 7 letters, respectively)*1), *or* (ii) $|\Delta|=379, 1379, 3404, 6671, 18529$ *or* 166754 *and* $G \neq S_{|\Delta|}, A_{|\Delta|}$.

In the present note we devote ourselves to the case where all orbits are self-paired. The remaining case where there exists non-self-paired orbit will be treated in a subsequent paper. There it will be shown that any 4-ply transitive permutation group (G, Δ) has no primitive extension of rank 4 $(\mathfrak{G}, \Omega)$ such that there exist non-self-paired orbits. Thus the determination of primitive extensions of rank 4 of 5-ply transitive permutation group is almost completed.

Our main idea of the proof of Theorem 1 is indebted to the concept of intersection matrices due to D. G. Higman [3], and is also indebted to some results of W. A. Manning (cf. P. J. Cameron [2]).

Just before this work has been done, S. Iwasaki has determined the pri-

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*) Supported in part by the Fujukai Foundation.

1) In these cases (G, Δ) have indeed primitive extensions of rank 4 $(\mathfrak{G}, \Omega)$ with regular normal subgroup of order 64.