J. Math. Soc. Japan
Vol. 25, No. 2, 1973

Some closed subalgebras of measure algebras and a generalization of P. J. Cohen's theorem II

By Jyunji INOUE

(Received Nov. 29, 1971)
(Revised Aug. 7, 1972)

§ 1. Introduction.

This paper is a continuation of the previous paper [4]. Throughout this paper $G(\tau)$ and $H(\sigma)$ denote LCA groups with underlying groups G and H, and with topologies τ and σ, respectively. In the previous paper [4], we introduced the closed subalgebra $L^*(G(\tau))$ of $M(G(\tau))$, and determined all the homomorphisms of $L^*(G(\tau))$ into $M(H(\sigma))$ as a generalization of Cohen's theorem. In this paper we prove that every homomorphism of $L^*(G(\tau))$ into $M(H(\sigma))$ has a natural norm-preserving extension to a homomorphism of $M(G(\tau))$ into $M(H(\sigma))$ as a generalization of Cohen's theorem.

In § 2 we give some preliminaries, and in § 3 we give the proof of our result for the special case that $H(\sigma)$ is compact. § 4 contains some results on the topology of the maximal ideal space of $M(G(\tau))$, which is used in § 5 to prove our result for the general case.

§ 2. Preliminaries.

We denote by $\mathfrak{T}(G(\tau))$ the set of all the locally compact group topologies on G which are at least as strong as the original topology τ. Let τ_1 and τ_2 be elements of $\mathfrak{T}(G(\tau))$ with $\tau_1 \subset \tau_2$. We denote by $\eta_{\tau_2}^{\tau_1}$ the natural continuous isomorphism of $G(\tau_2)$ onto $G(\tau_1)$. Γ_{τ_i} denotes the dual group of $G(\tau_i)$ and $\varphi_{\tau_2}^{\tau_1}$ denotes the natural continuous isomorphism of Γ_{τ_1} onto a dense subgroup of Γ_{τ_2} such that (cf. Lemma 2.3 of [4])

$$(x, \varphi_{\tau_2}^{\tau_1}(r)) = (\eta_{\tau_2}^{\tau_1}(x), r) \qquad (x \in G(\tau_2),\ r \in \Gamma_{\tau_1}).$$

For each $\tau' \in \mathfrak{T}(G(\tau))$, there exists a natural norm-preserving isomorphism $\pi_{\tau'}$ of $M(G(\tau'))$ into $M(G(\tau))$ such that (cf. Proposition 2.1 of [4])

$$\pi_{\tau'}(\mu)(E) = \mu(\eta_{\tau'}^{\tau\,-1}(E)) \qquad (E\text{: Borel set of } G(\tau)\,;\ \mu \in M(G(\tau'))).$$

We identify $L^1(G(\tau'))$ and $M(G(\tau'))$ with the closed subalgebras of $M(G(\tau))$ through $\pi_{\tau'}$, respectively. $M(G(\tau'))^{\perp} = \{\mu \in M(G(\tau)) : \mu \perp \nu\,;\ \nu \in M(G(\tau'))\}$ is an